Concurrent Skip Lists

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Based on slides by Maurice Herlihy and Nir Shavit

Set Object Interface

- Collection of elements
- No duplicates
- Methods
  - add() a new element
  - remove() an element
  - contains() if element is present

Expectation

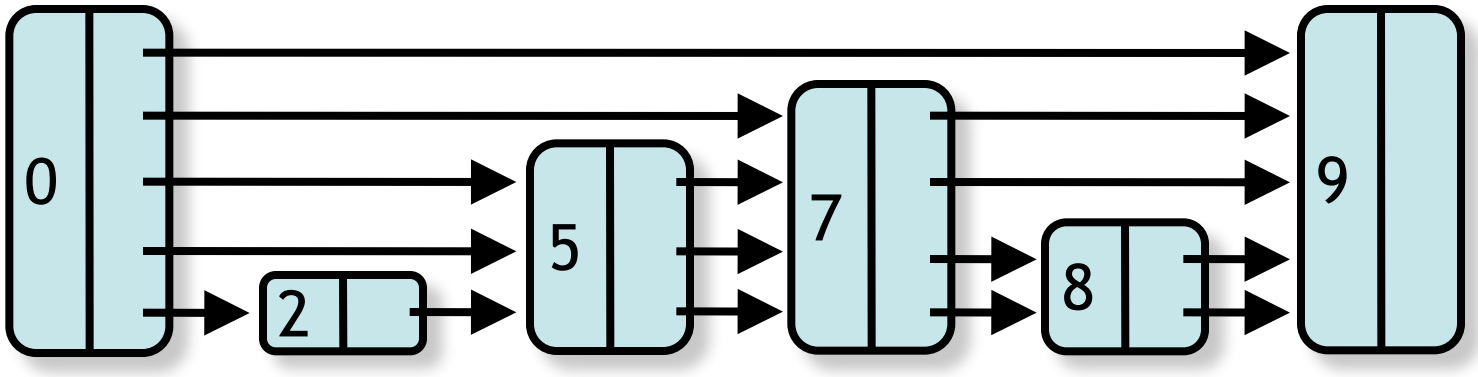
- Typically high % of contains() calls
- Many fewer add() calls
- And even fewer remove() calls
  - 90% contains()
  - 9% add()
  - 1% remove()
- Folklore?
  - Yes but probably mostly true

Concurrent Sets

- Balanced Trees?
  - Red-Black trees, AVL trees, …
- Problem: no one does this well …
- … because **rebalancing** after add() or remove() is a global operation

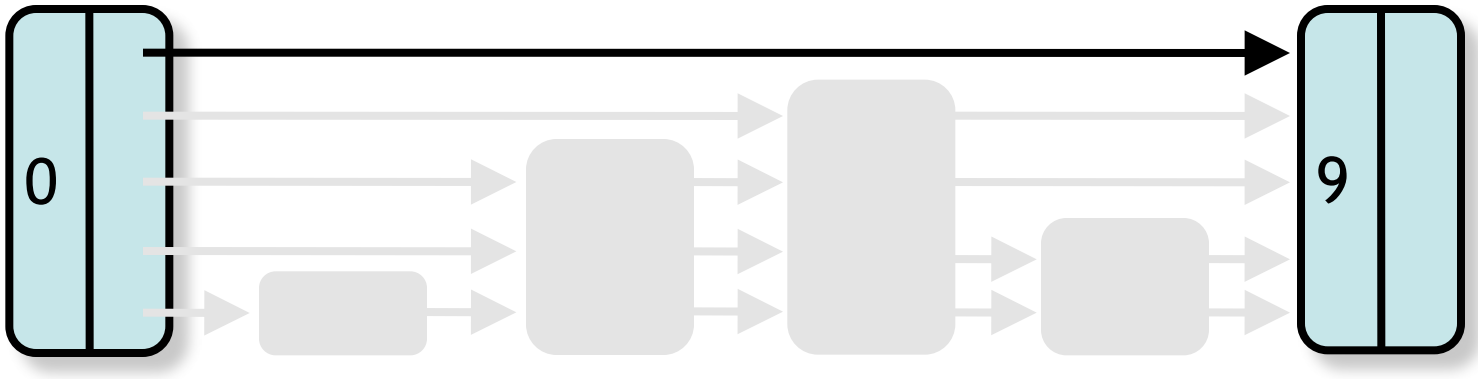
Skip Lists

- Probabilistic Data Structure
- No global rebalancing
- Logarithmic-time search



Skip List Property

- Each layer is sublist of lower-levels



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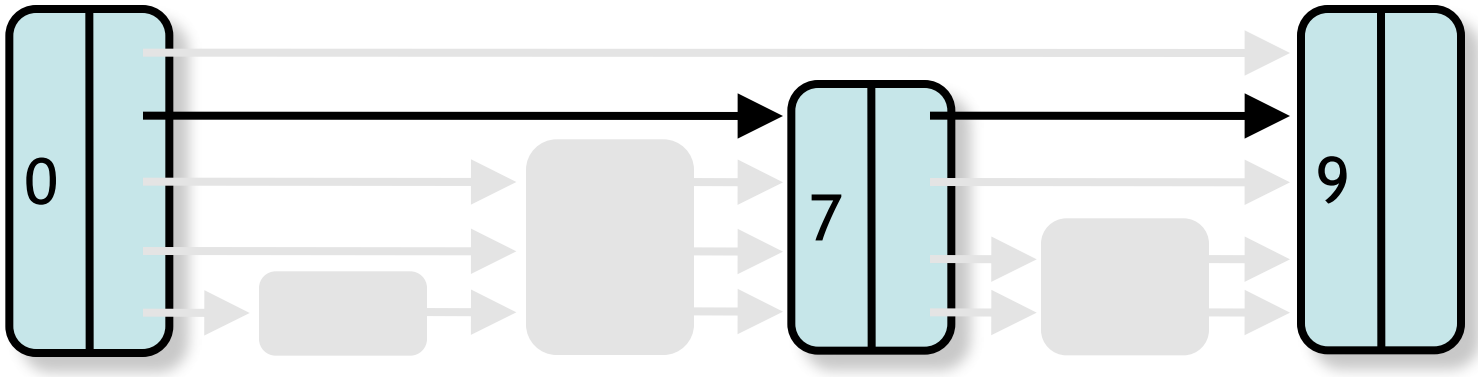

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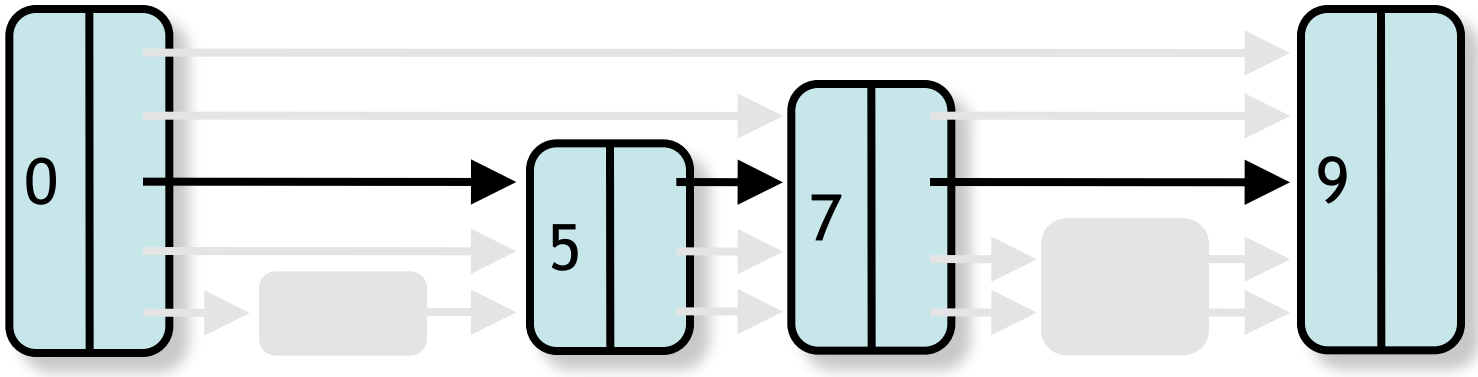

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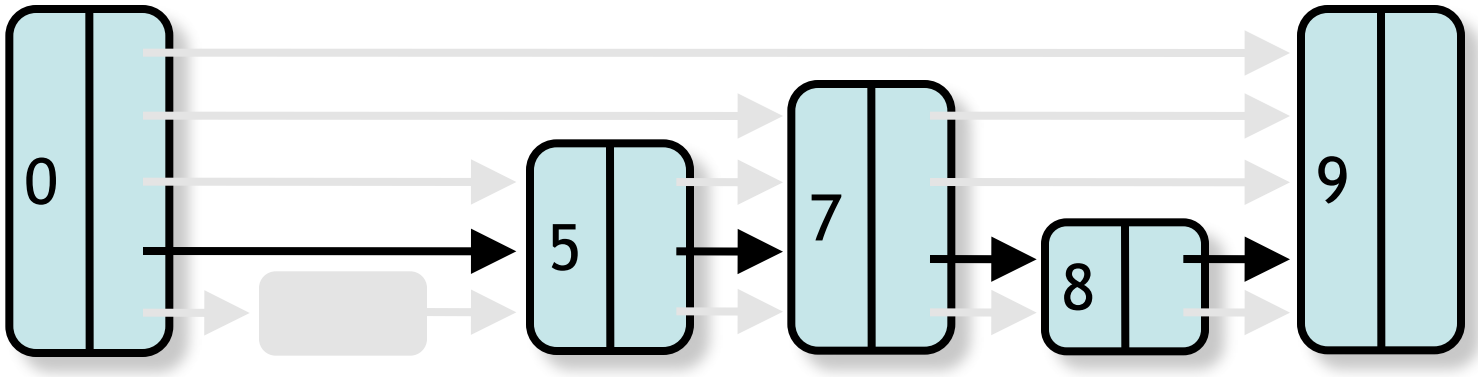
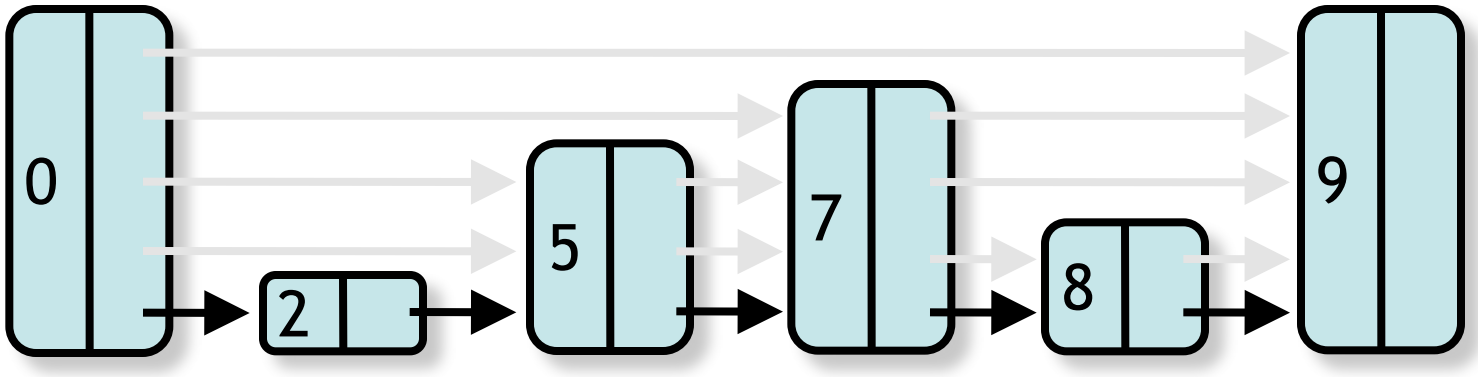


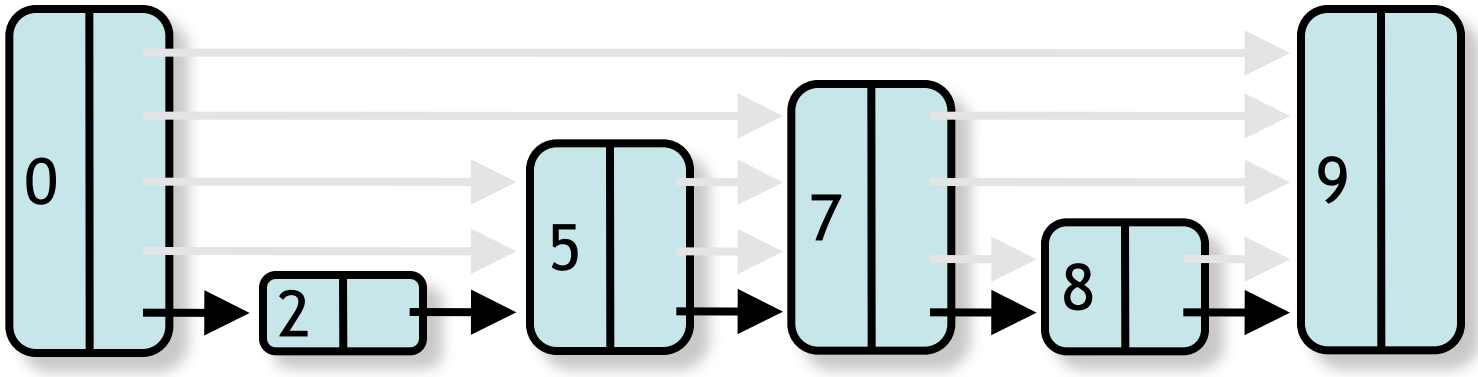
Skip List Property

- Each layer is sublist of lower-levels
- Lowest level is entire list



Skip List Property

- Each layer is sublist of lower-levels
- Not easy to preserve in concurrent implementations …



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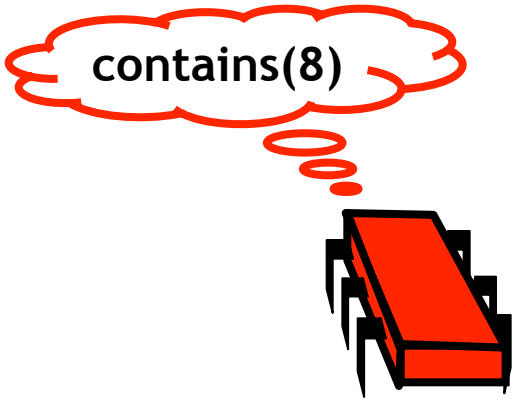

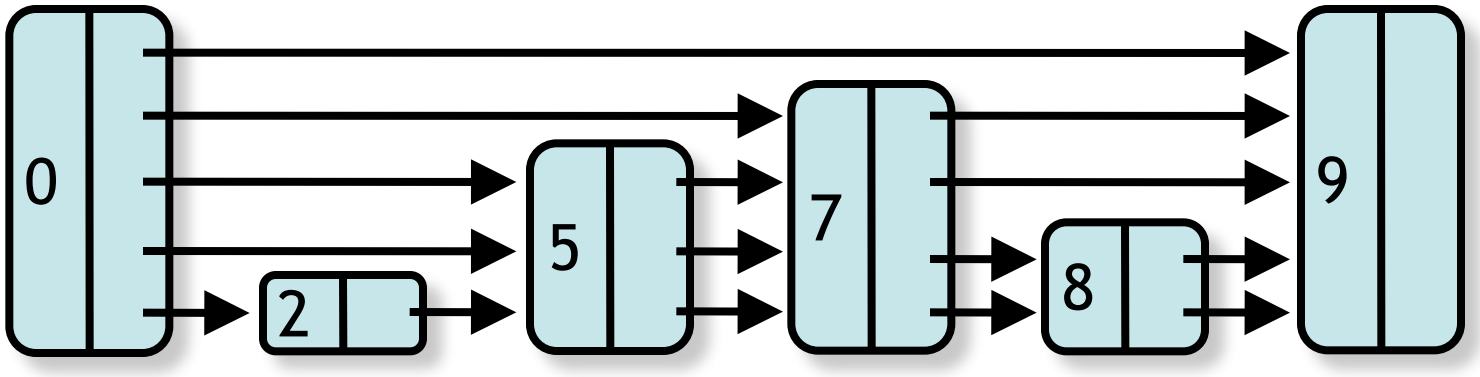

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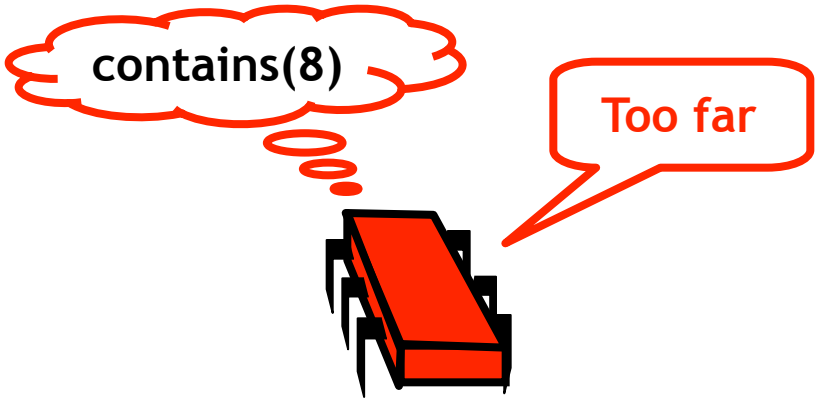

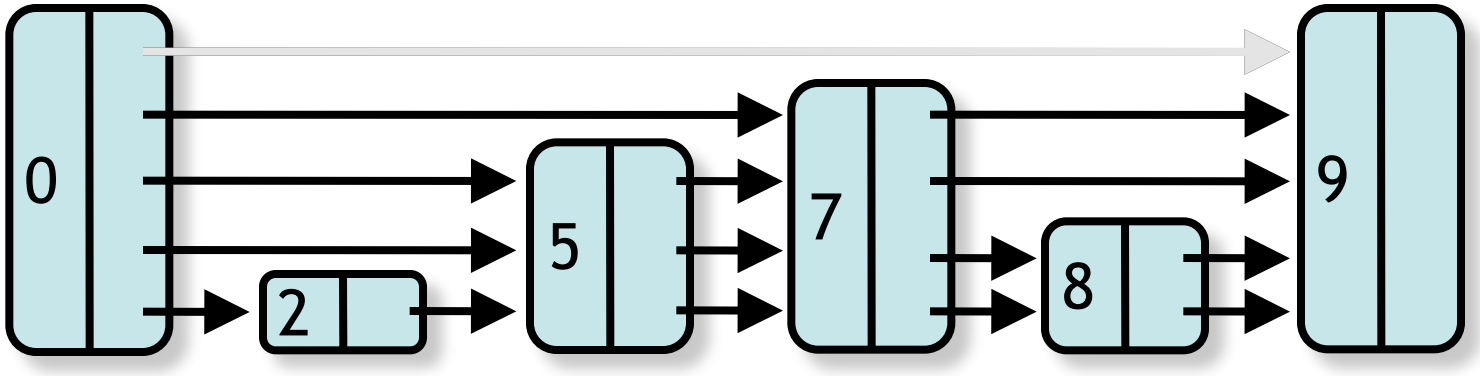

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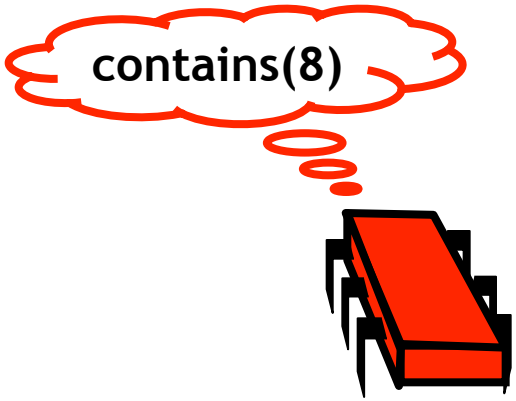

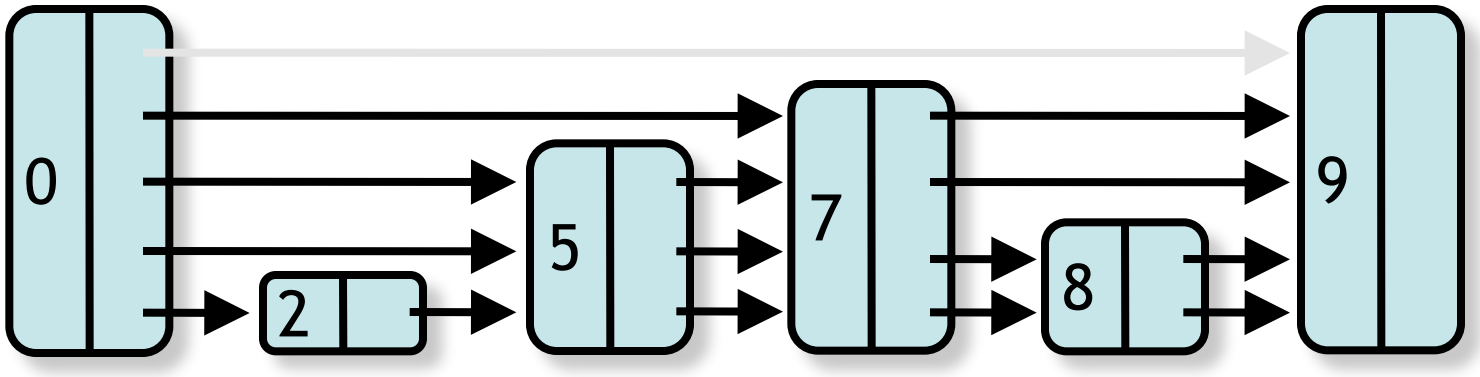

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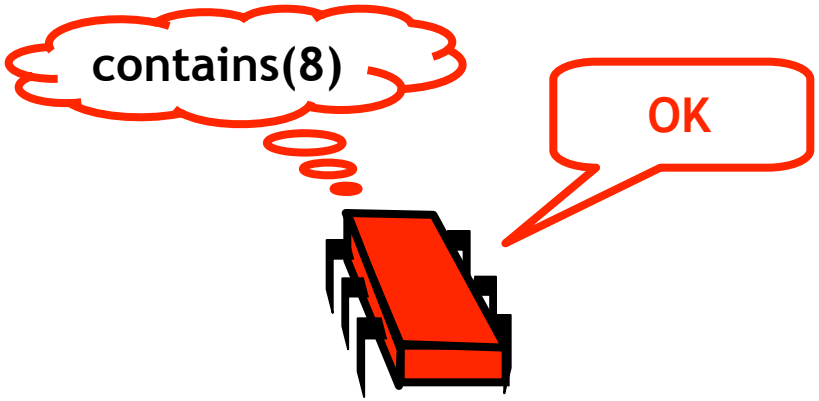

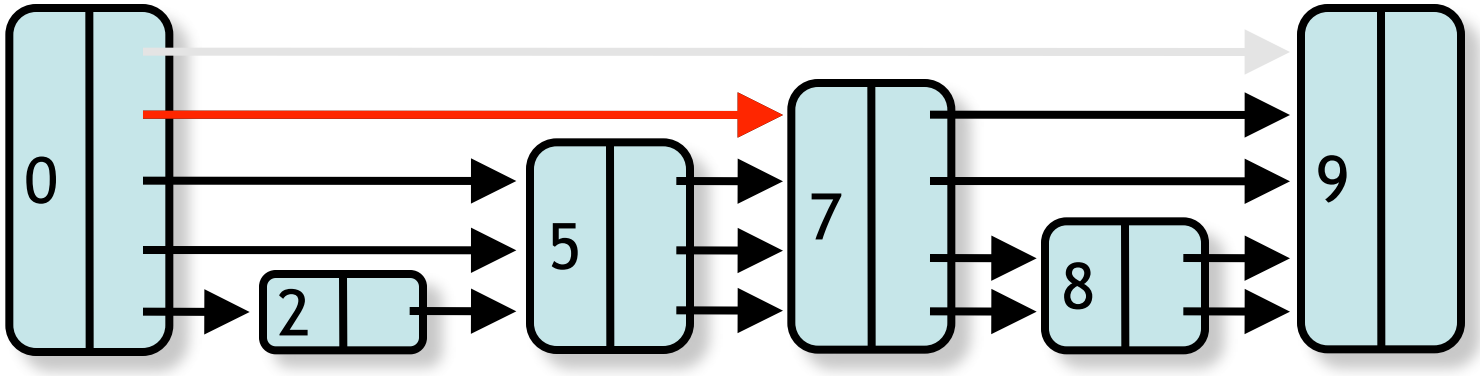

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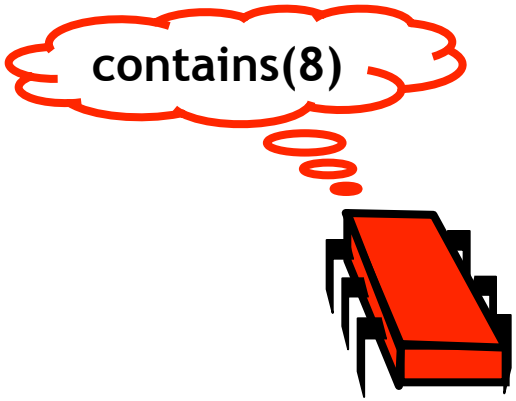

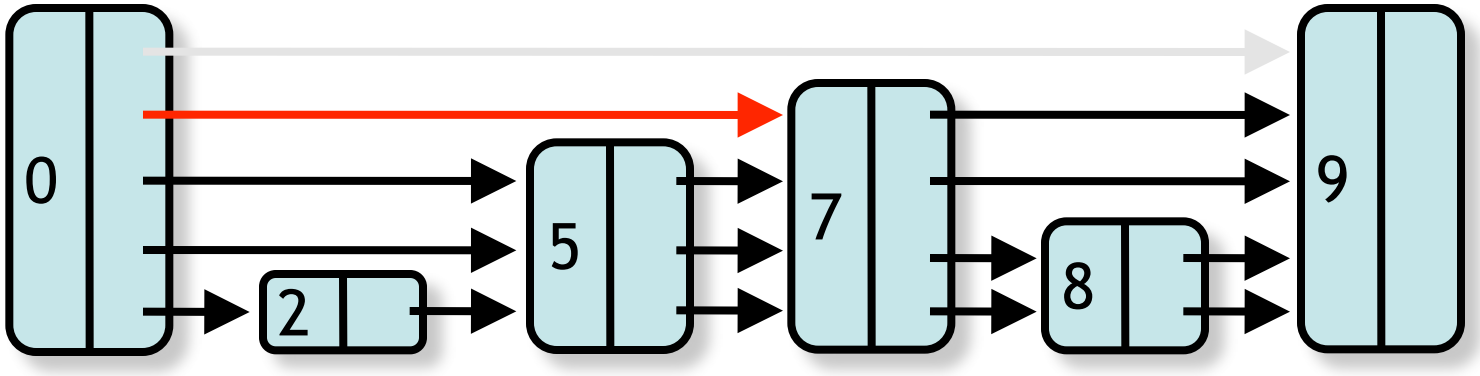

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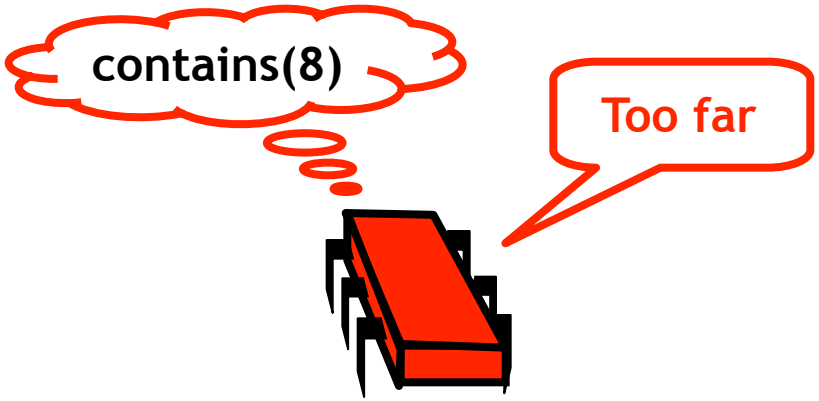

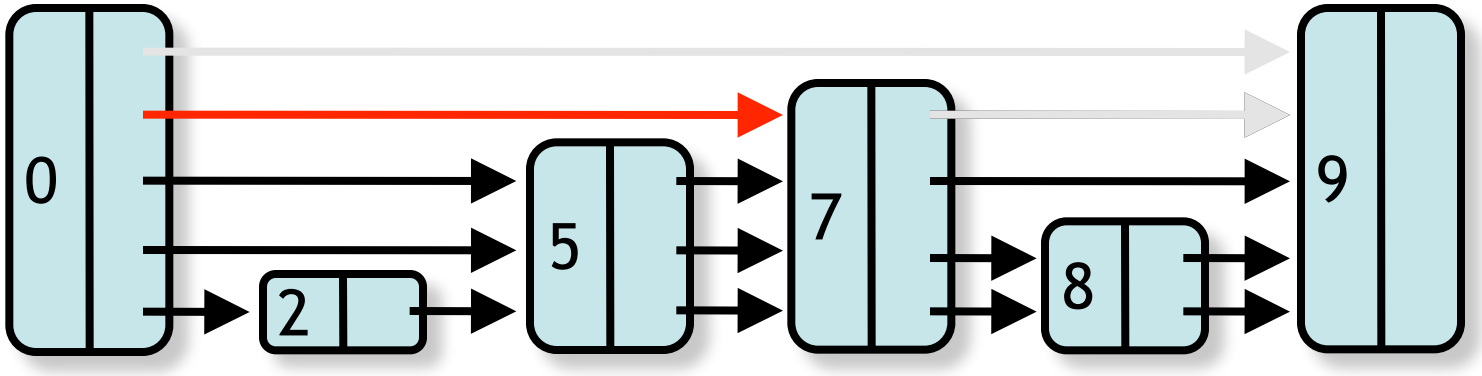

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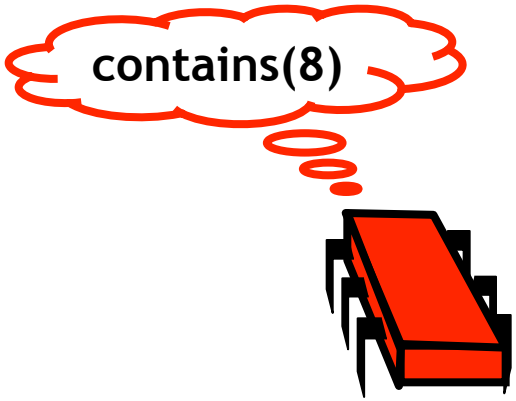

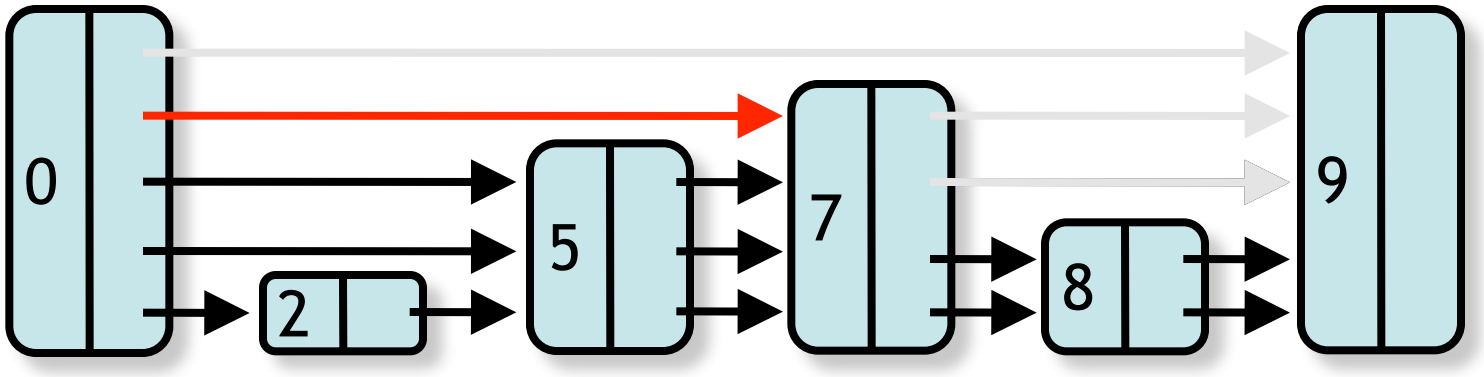

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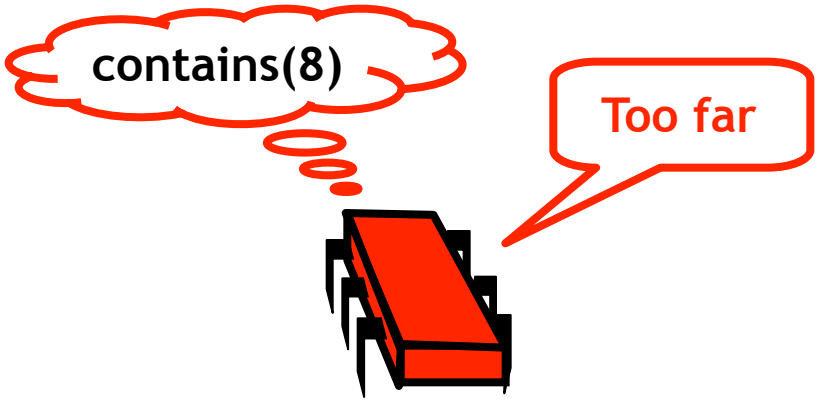

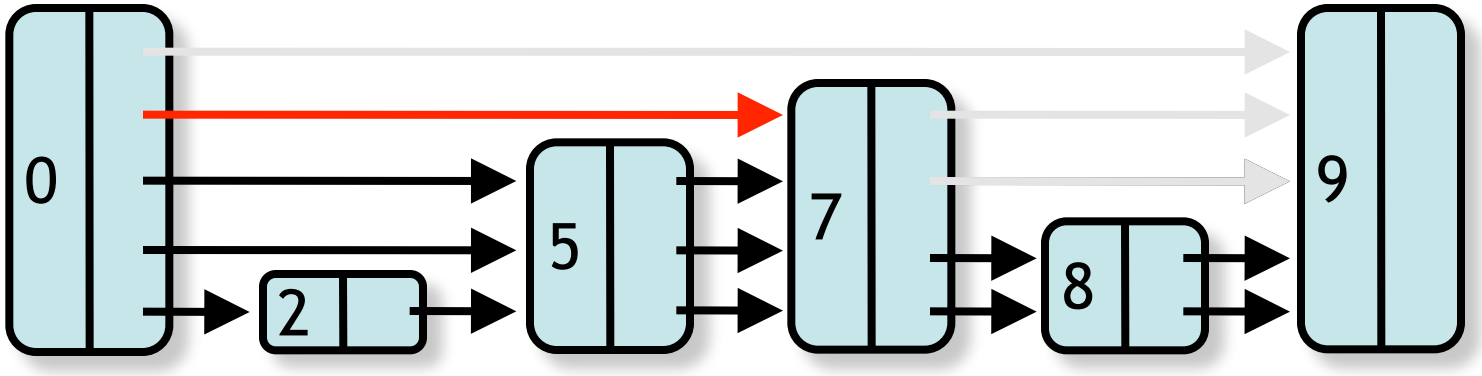

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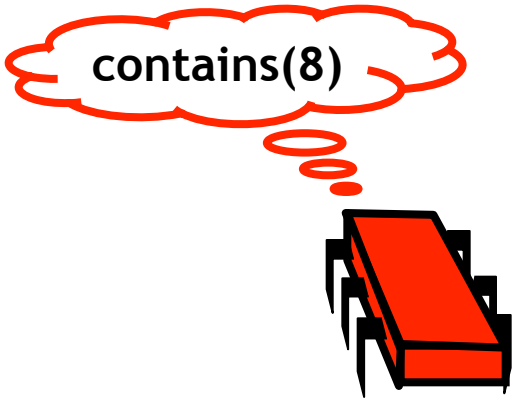

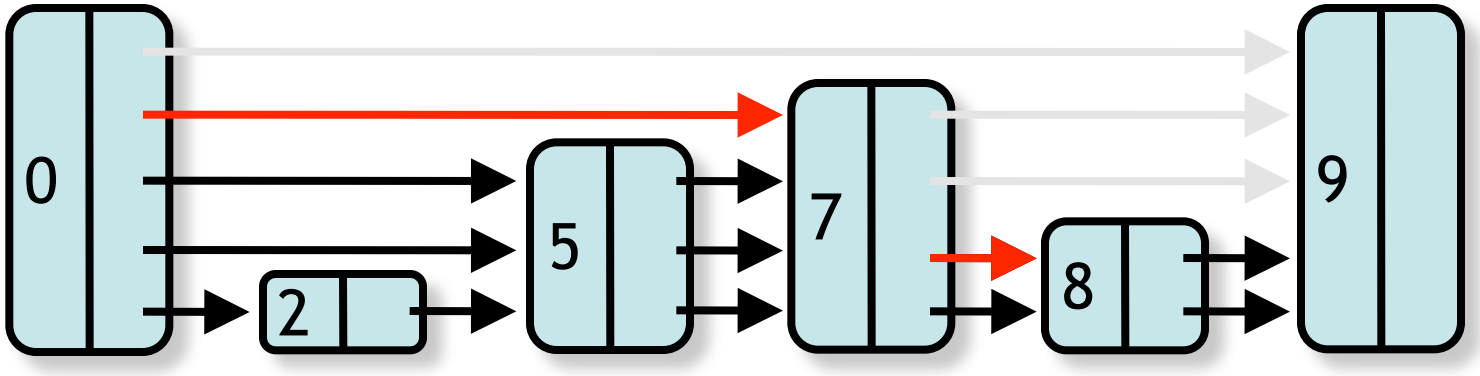

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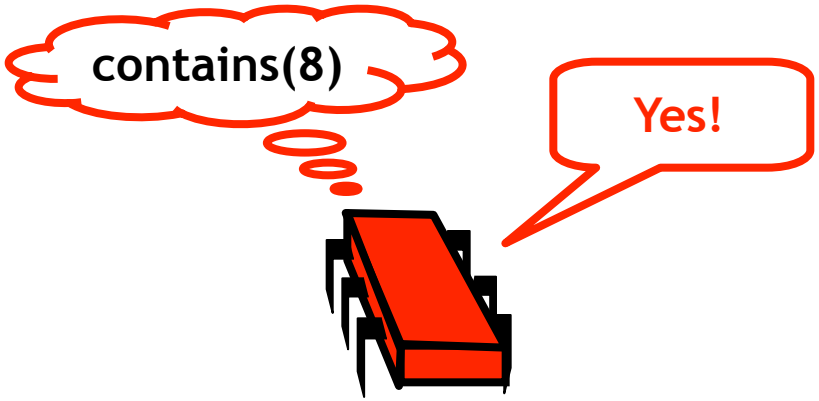

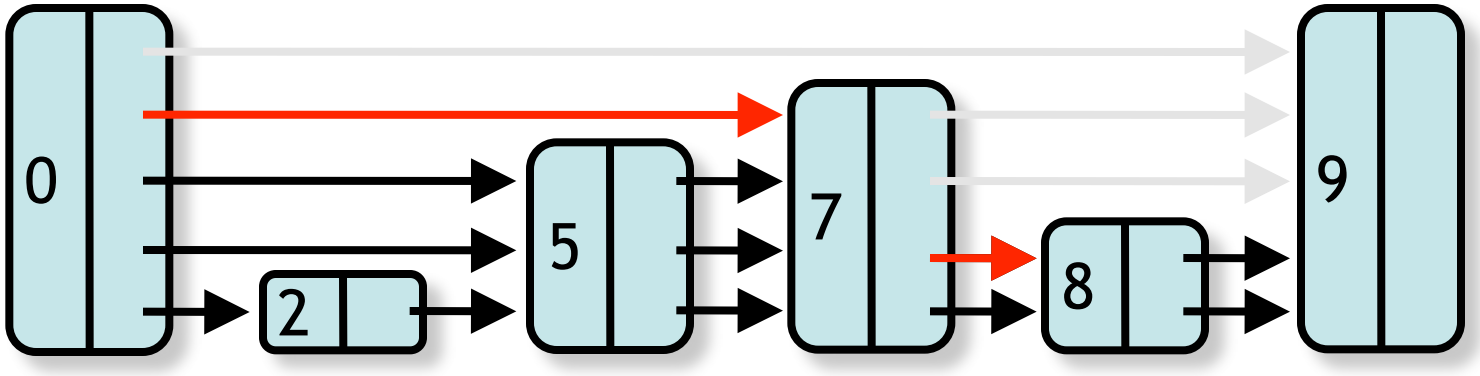

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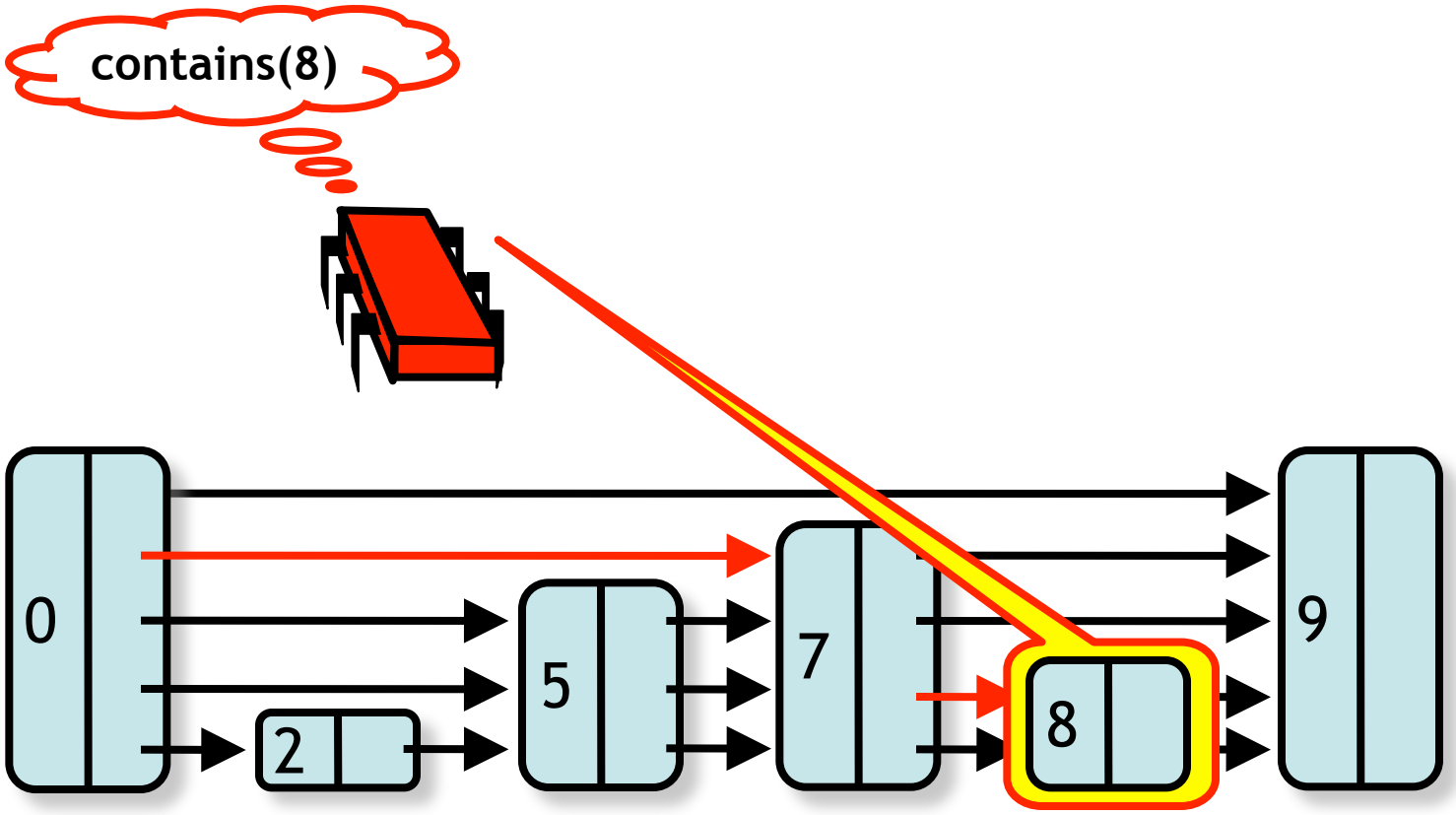
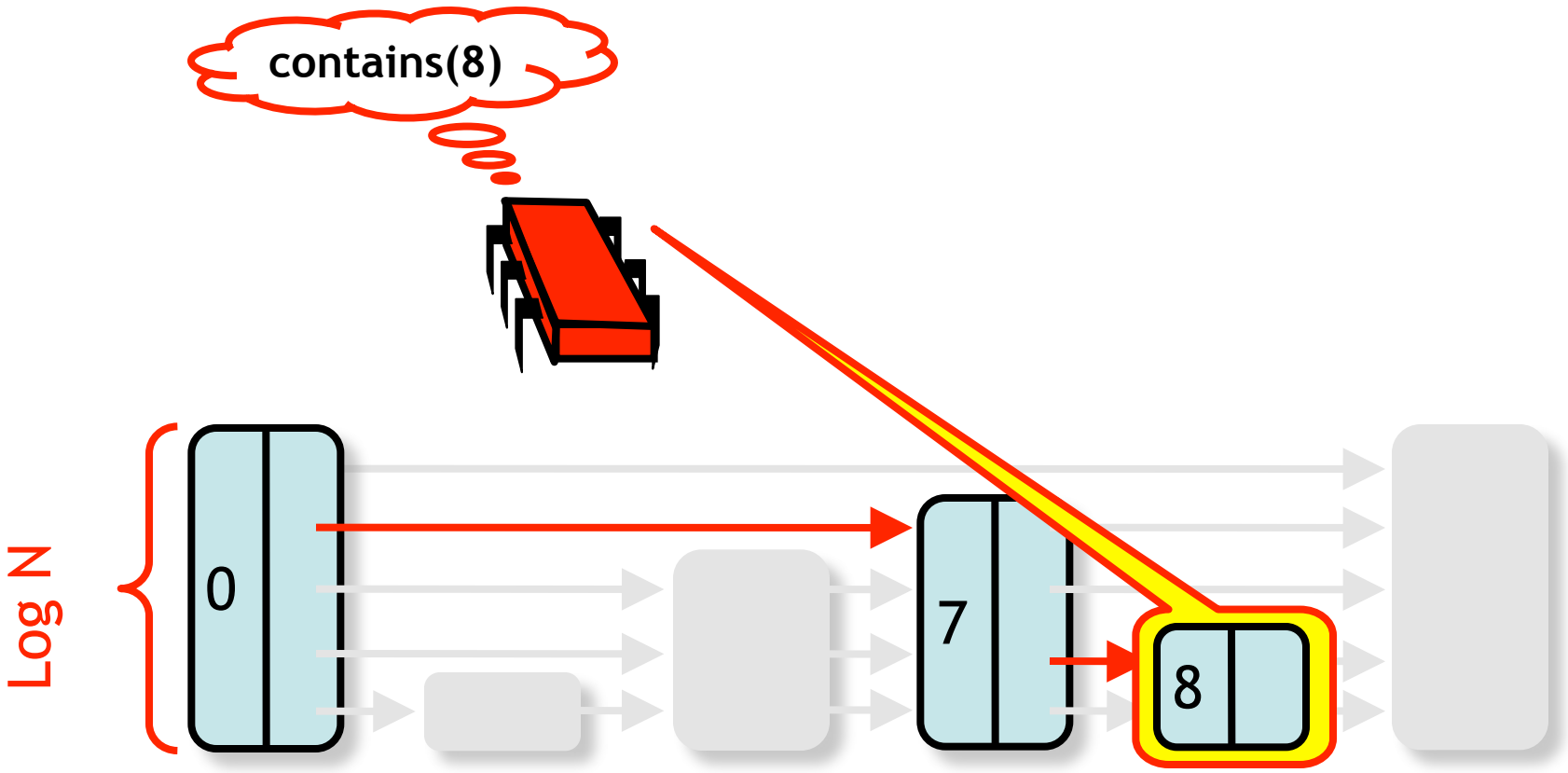
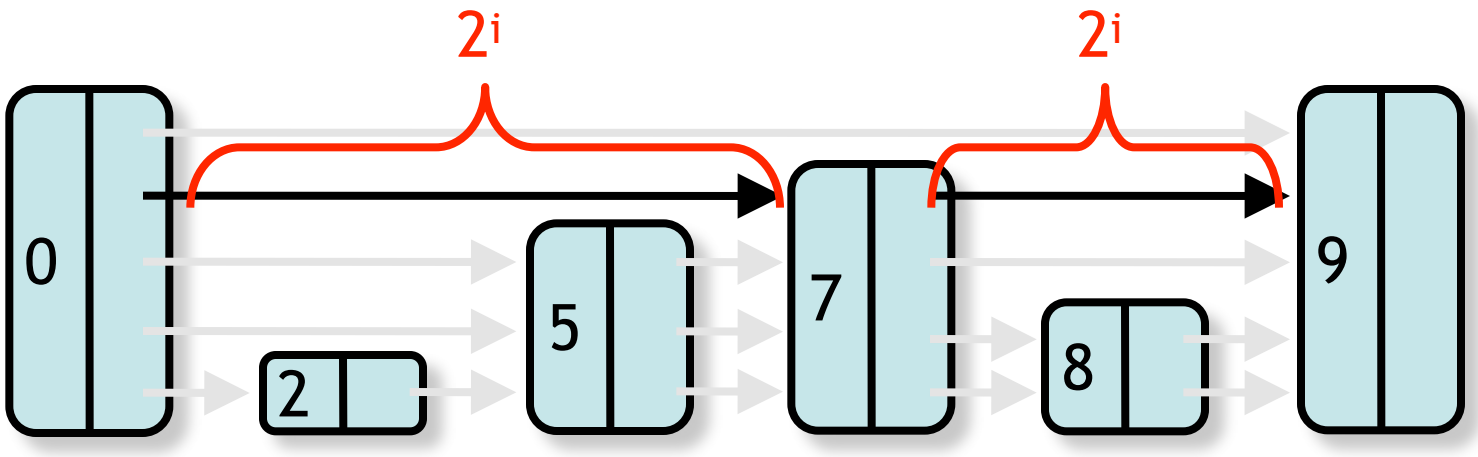


Logarithmic



Why Logarthimic

- Property: Each pointer at layer i jumps over roughly 2^i nodes
- Pick node heights randomly so property guaranteed probabilistically



Find() -- Sequential

```
int find(T x, Node<T>[] preds, Node<T>[] succs) {
  …
```

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int find(T x, Node<T>[] preds, Node<T>[] succs) {
  ...
```

object height
(-1 if not there)

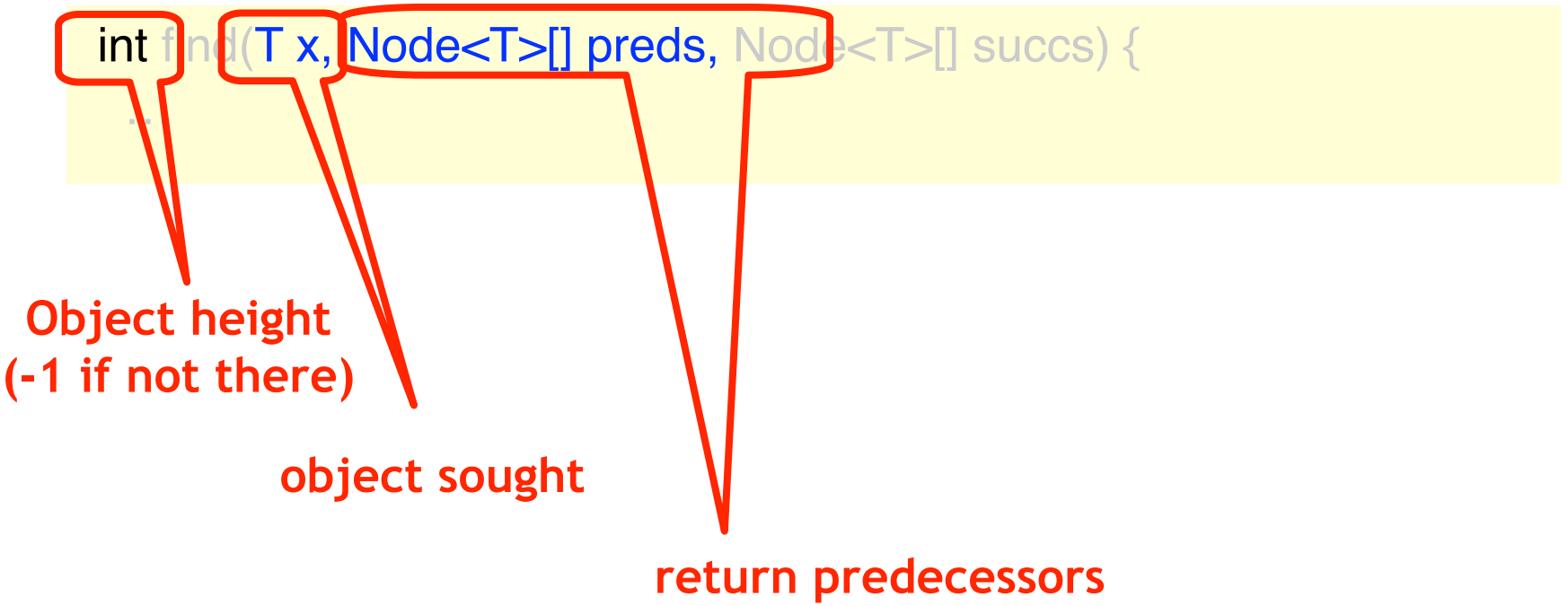
Find() -- Sequential

```
int find(T x, Node<T>[] preds, Node<T>[] succs) {
  ...
```

object height (-1 if not there)

Object sought

Find() -- Sequential



Find() -- Sequential

```
int find(T x, Node<T>[] preds, Node<T>[] succs) {
  . . .
```

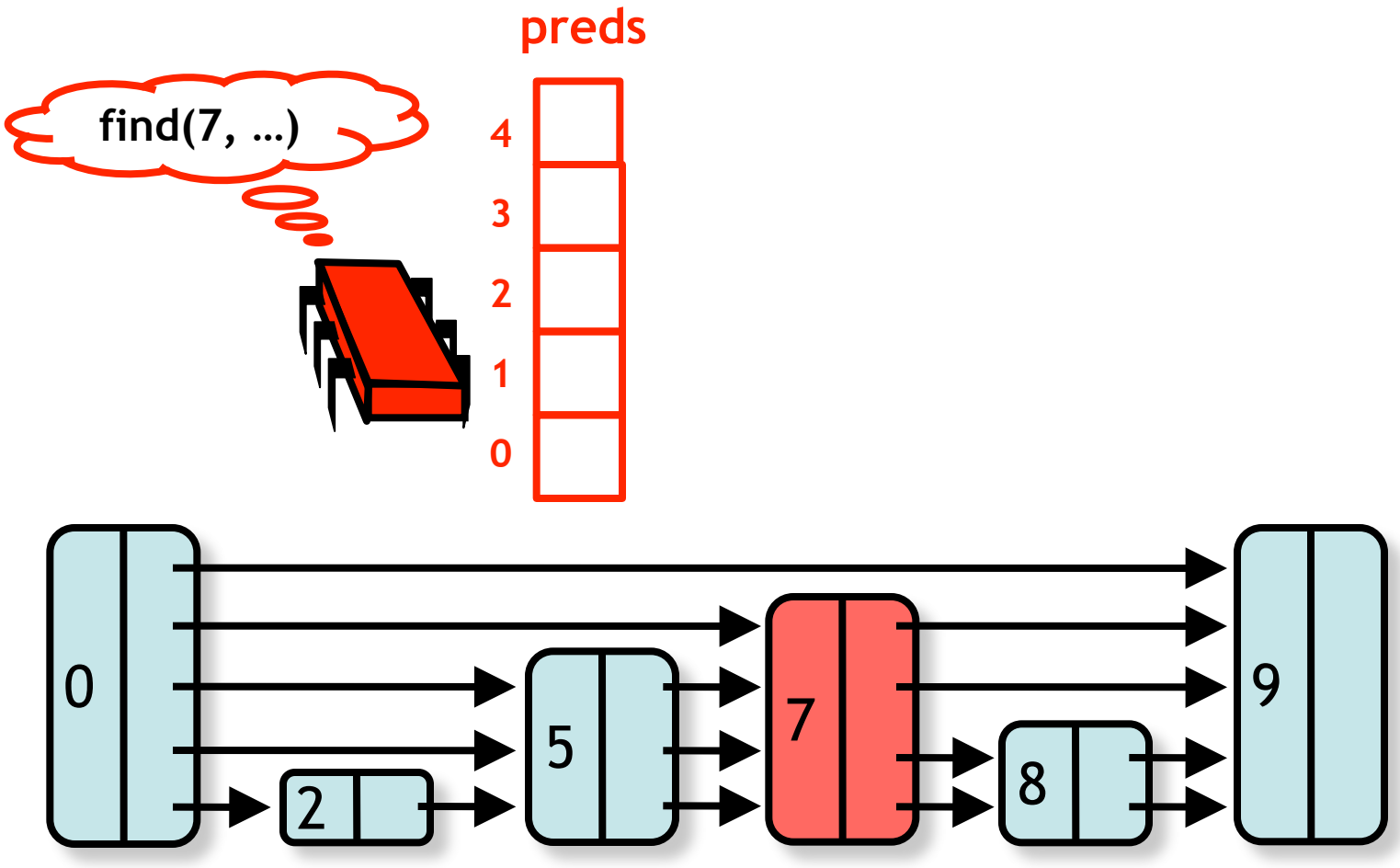
object height (-1 if not there)

object sought

return predecessors

return successors

Successful Search



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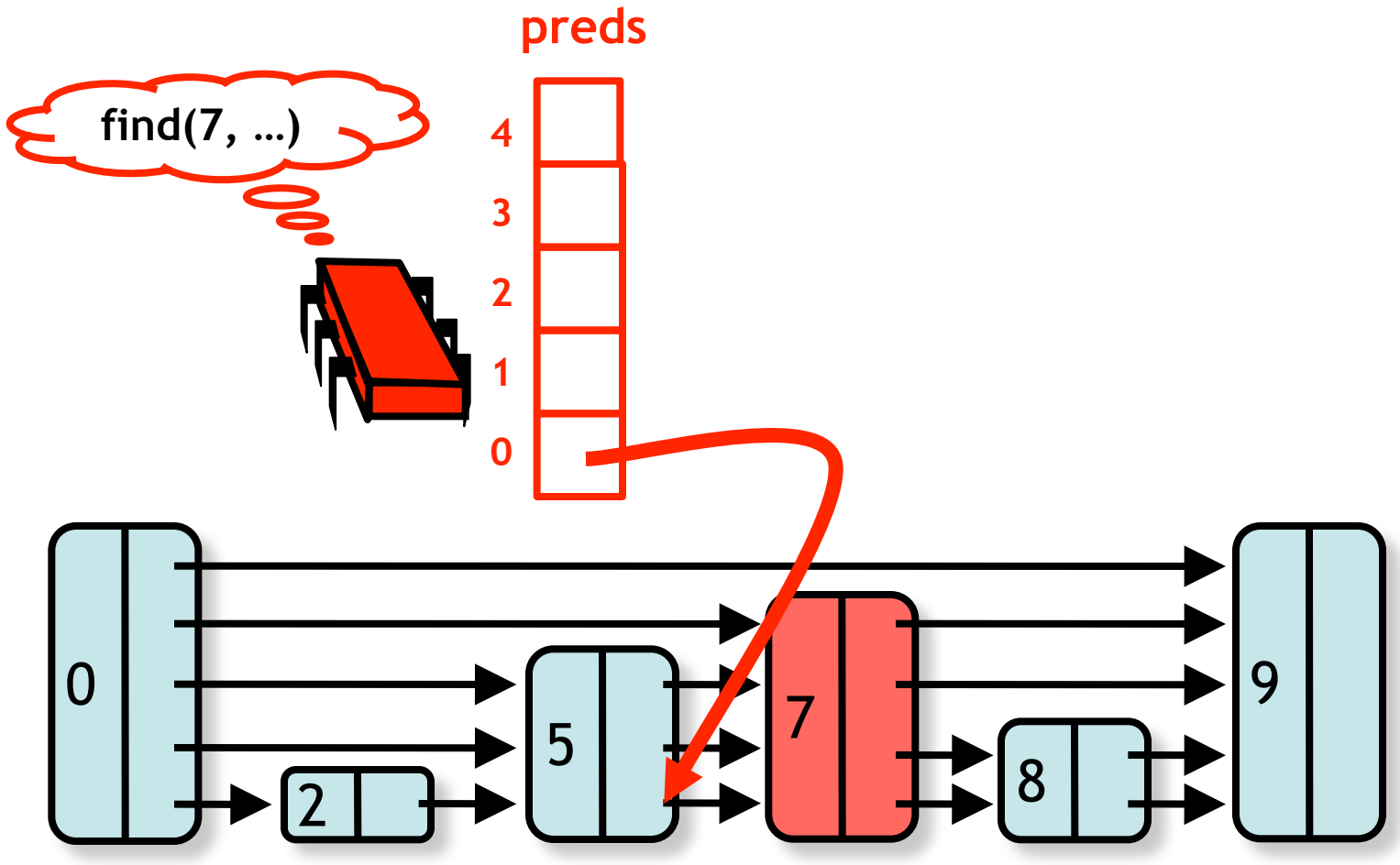

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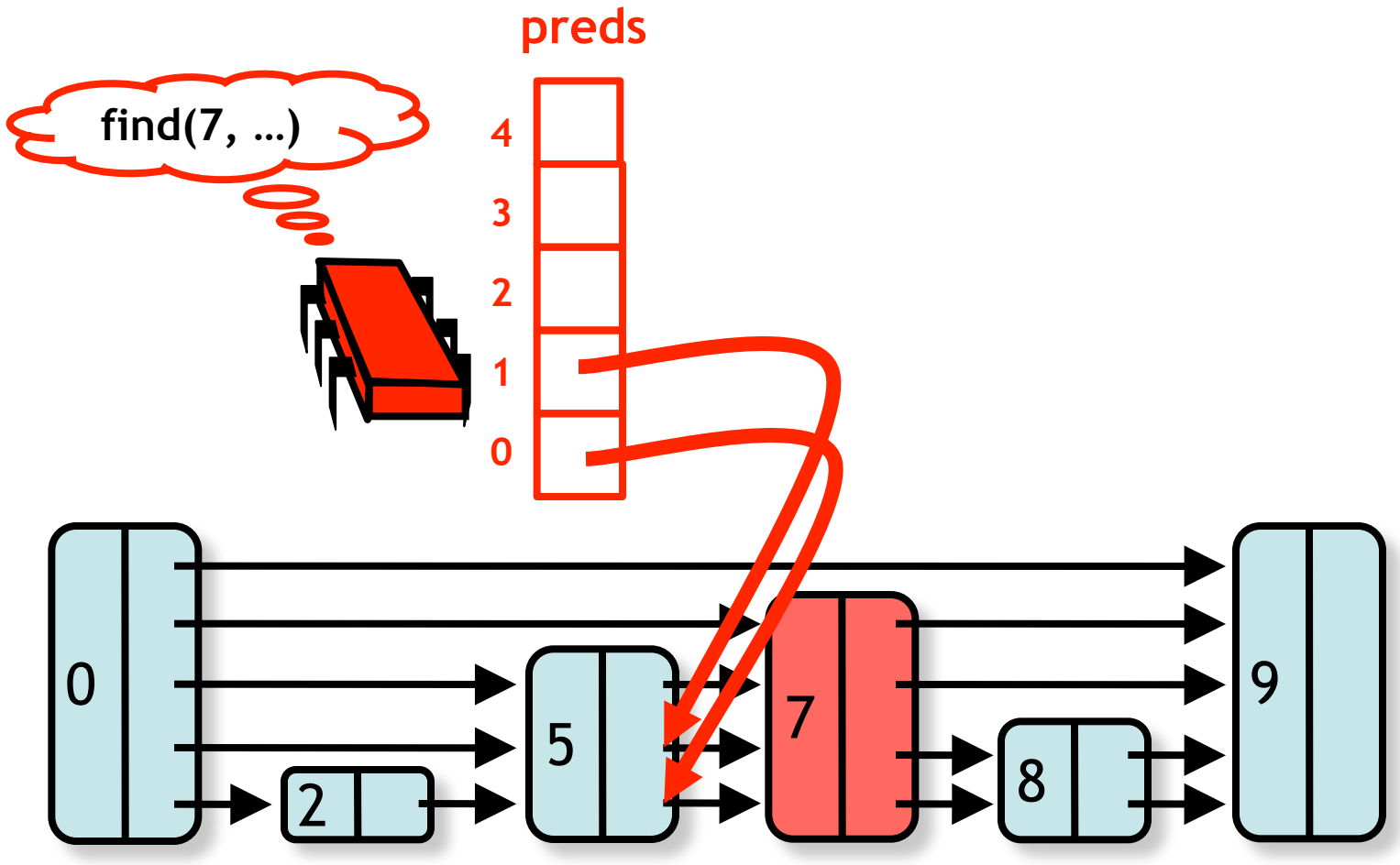

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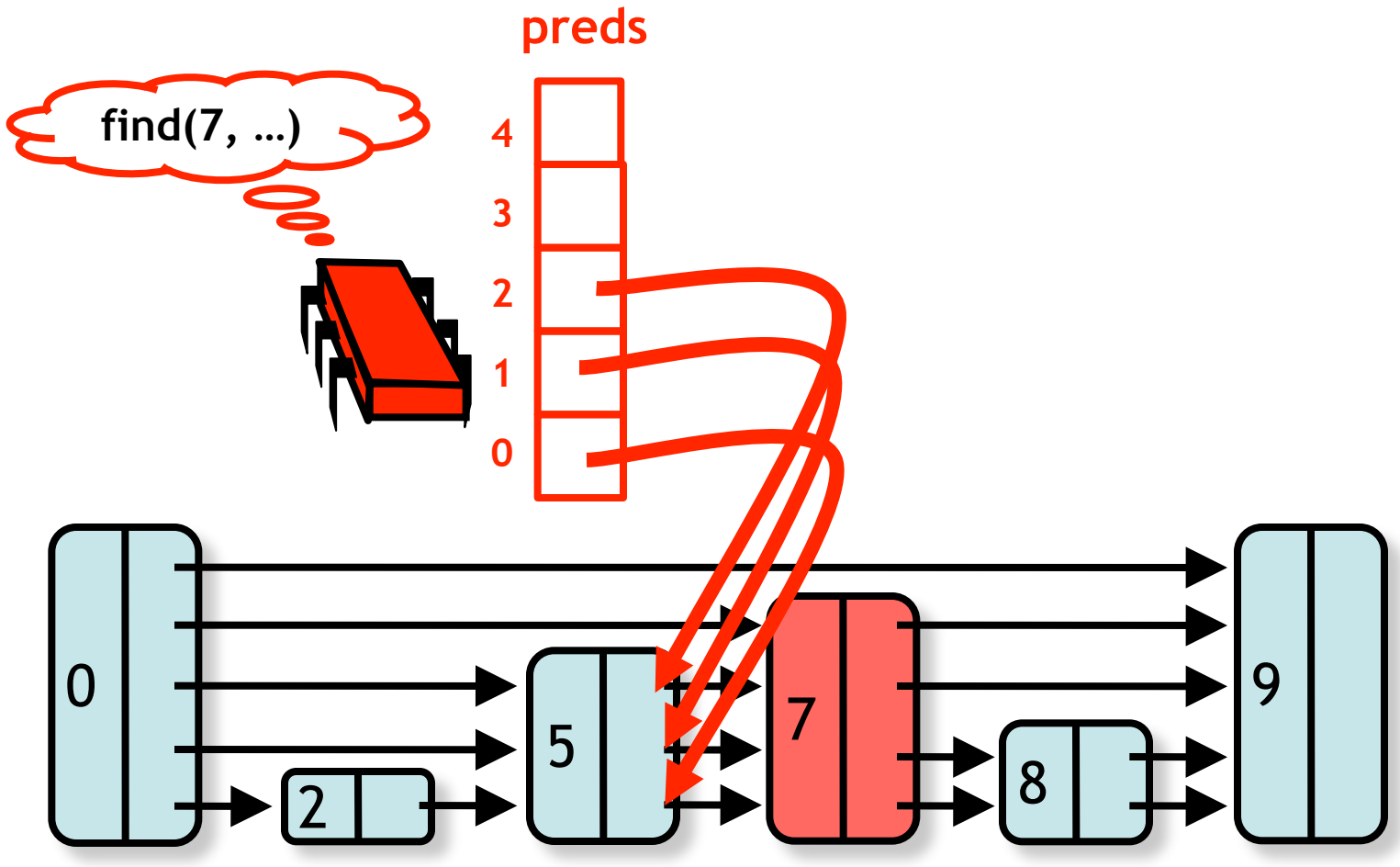

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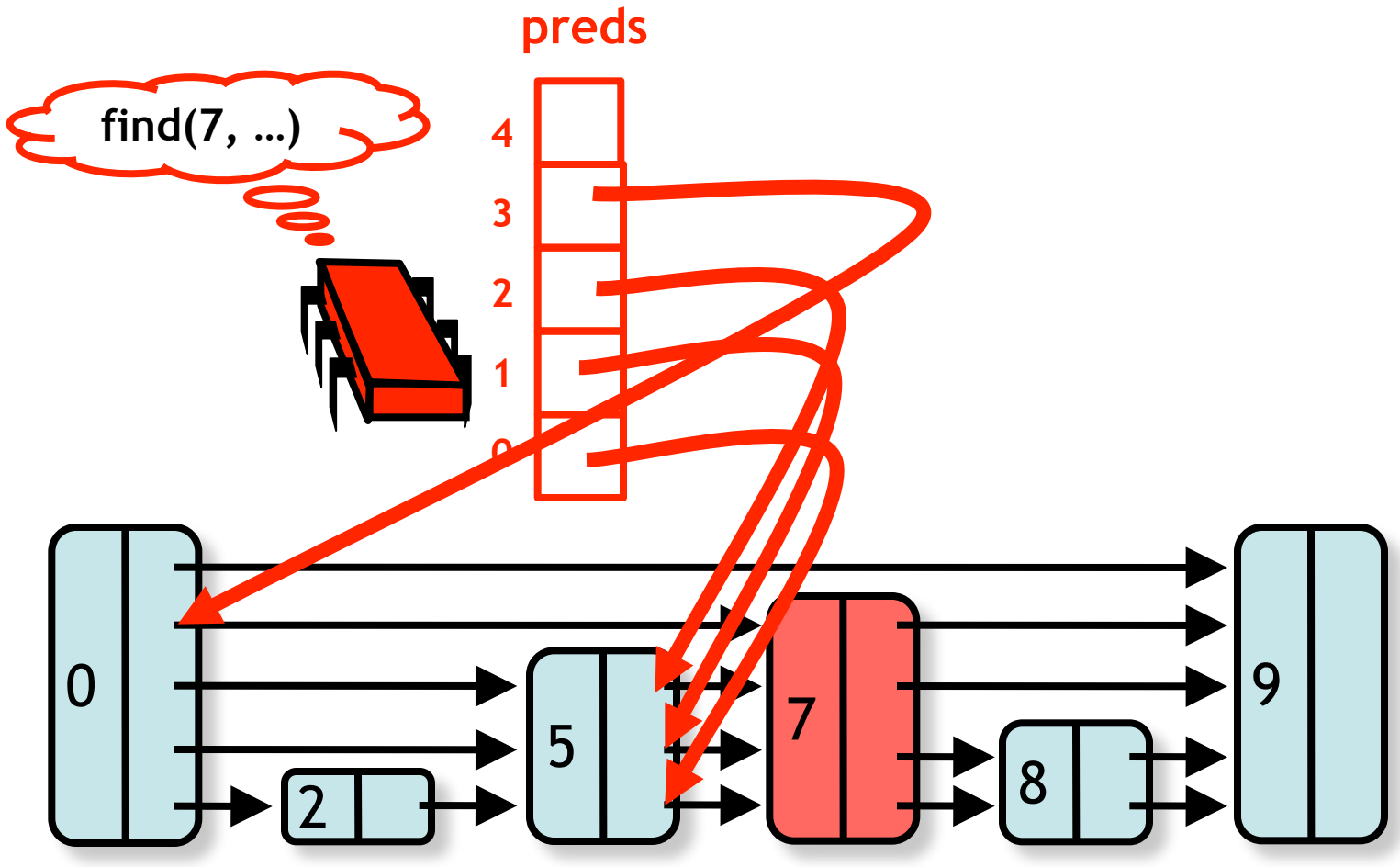

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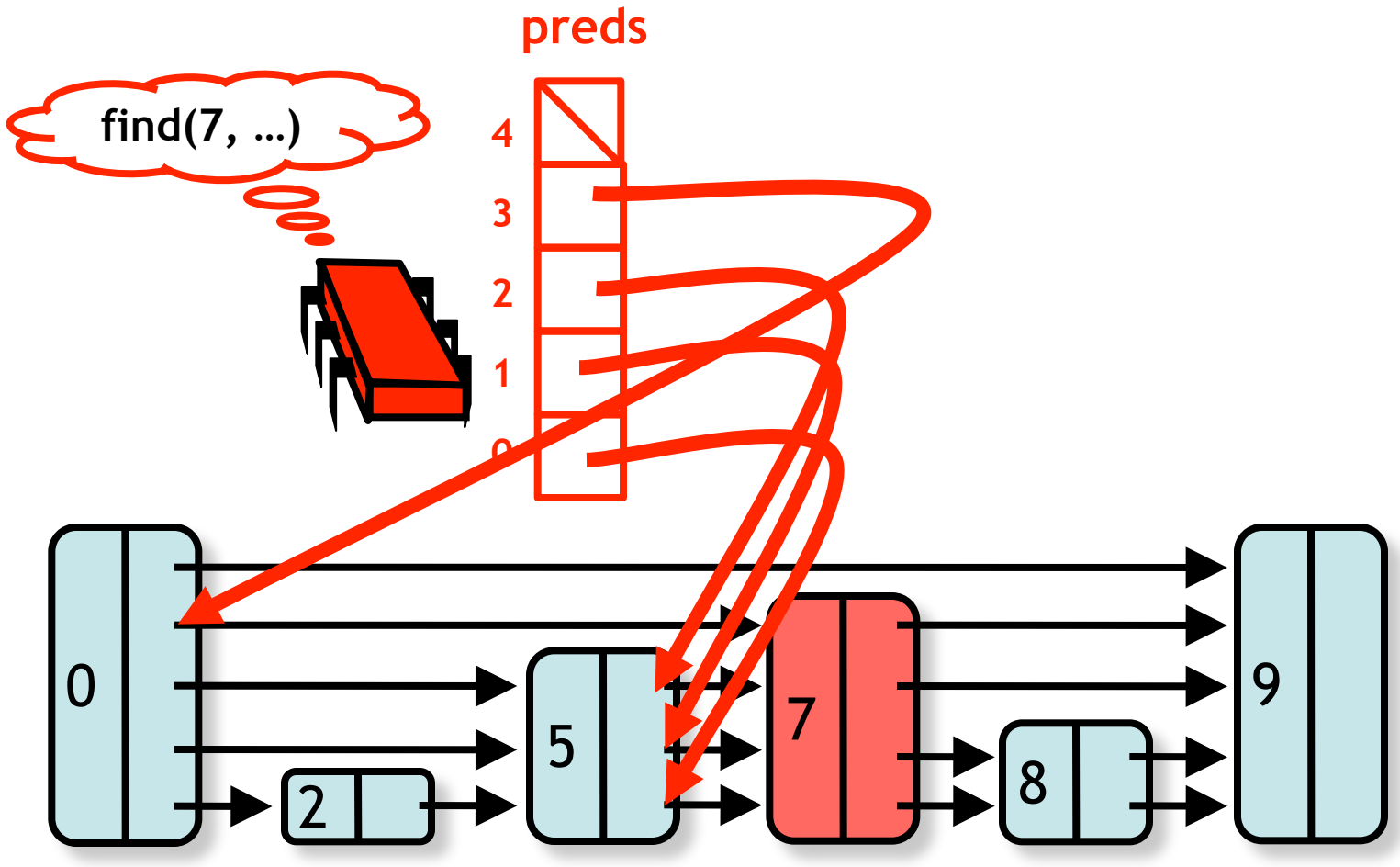

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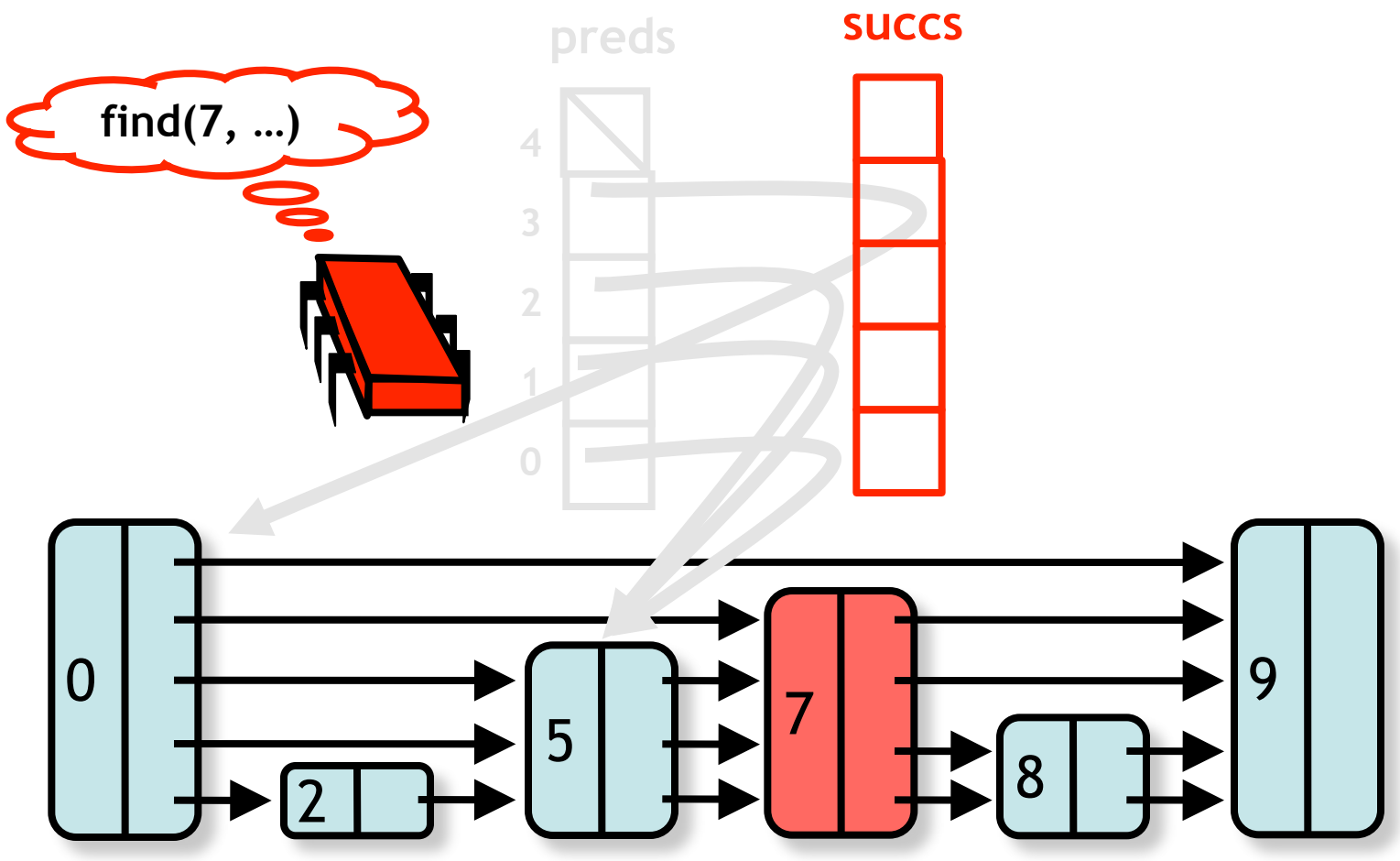

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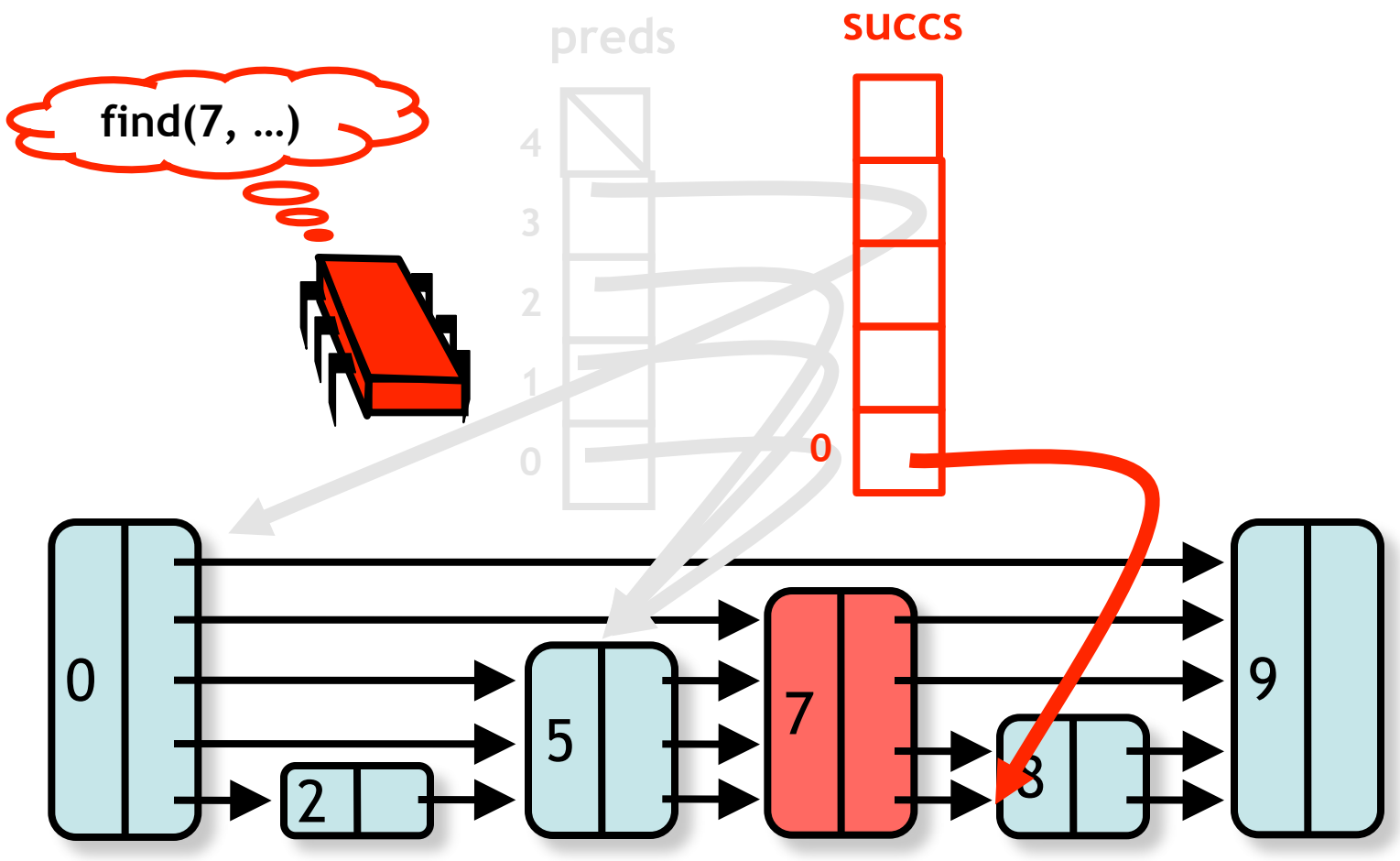

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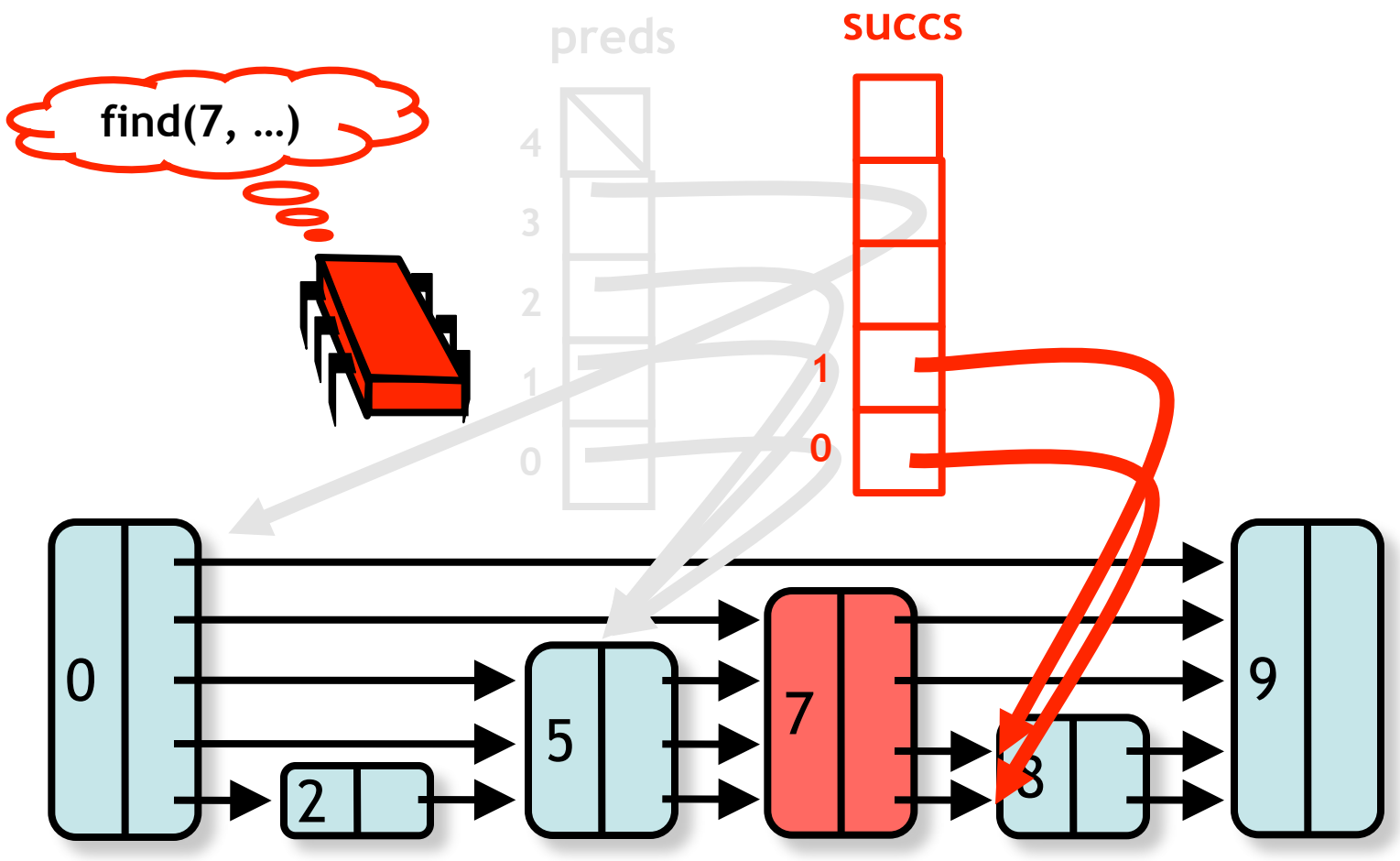

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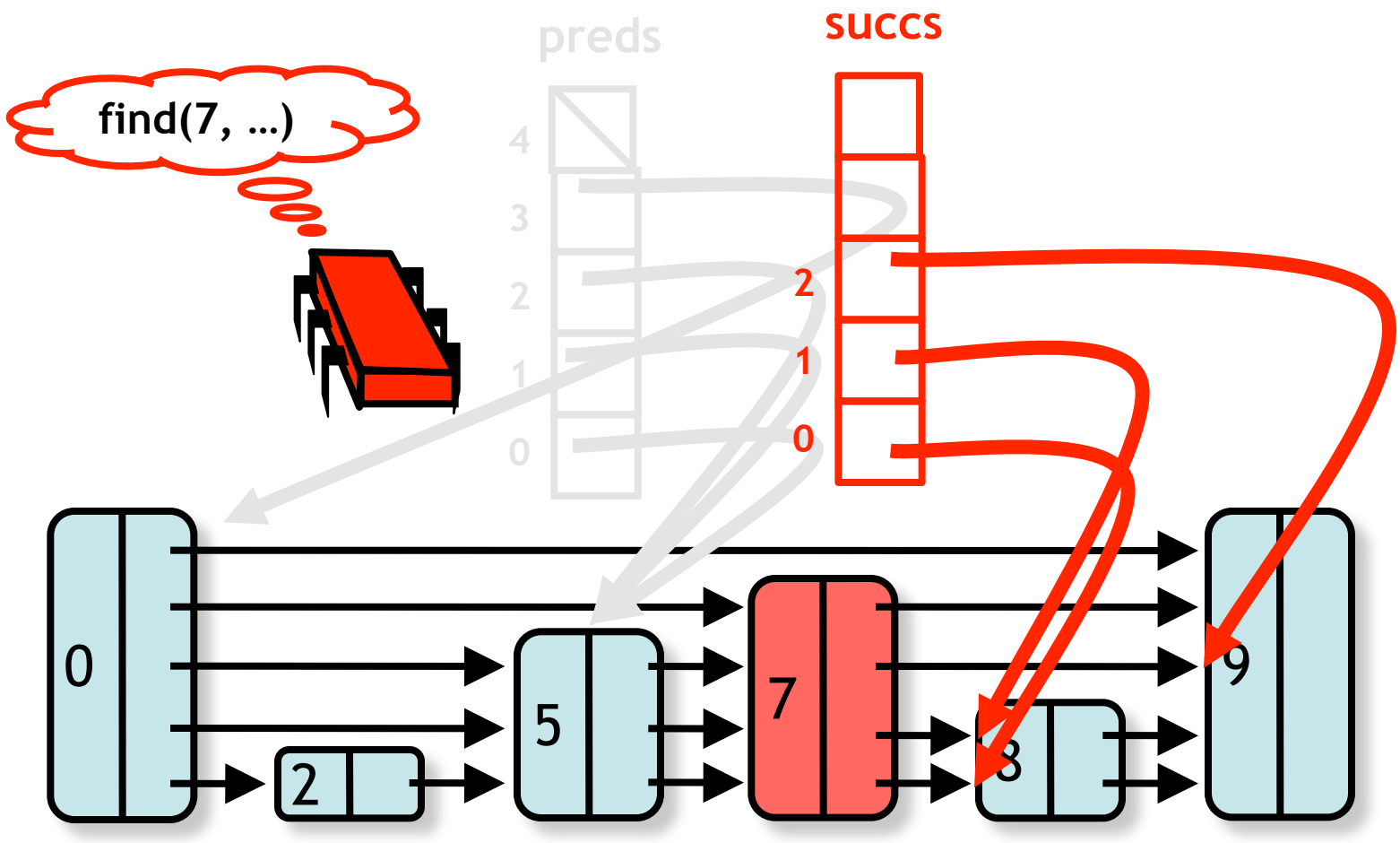

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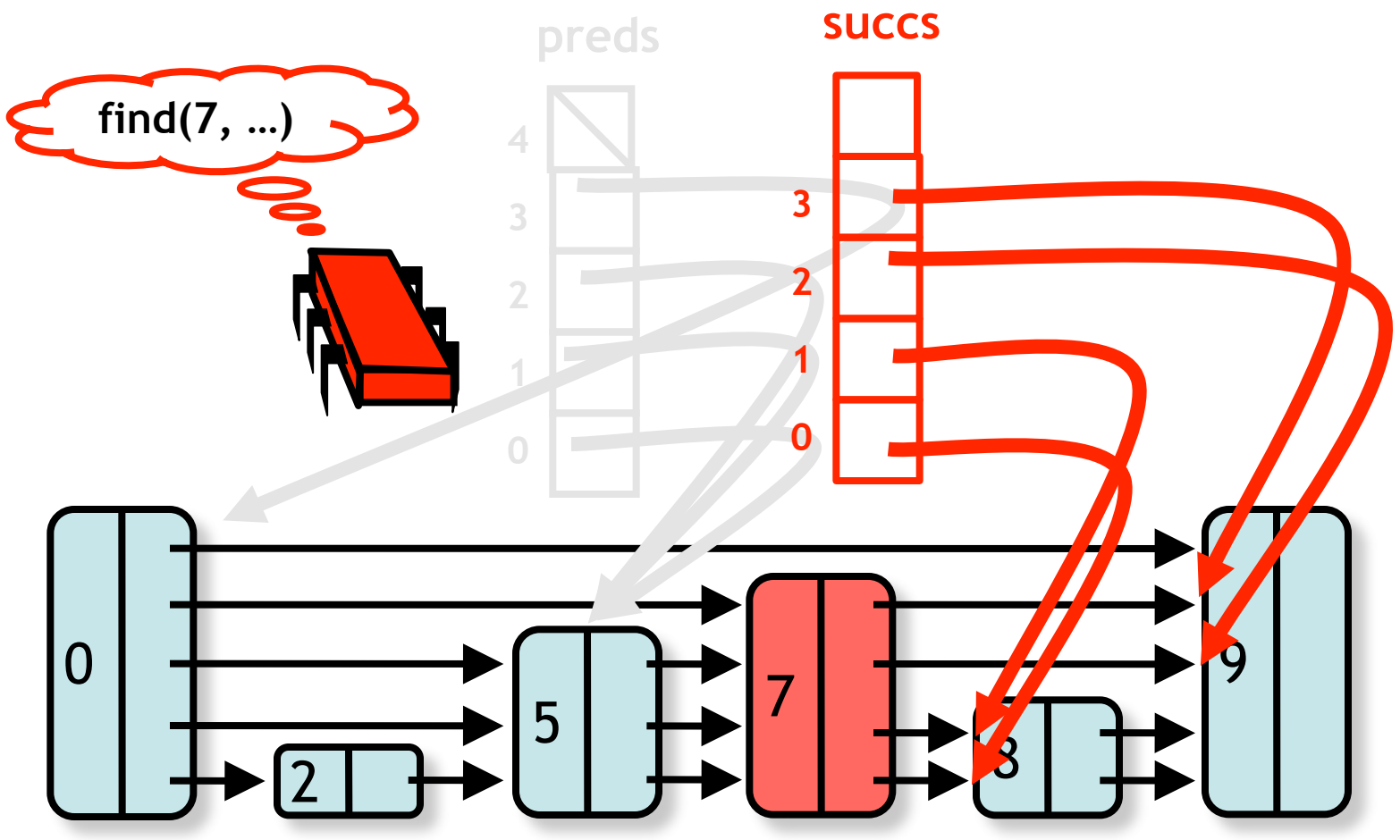

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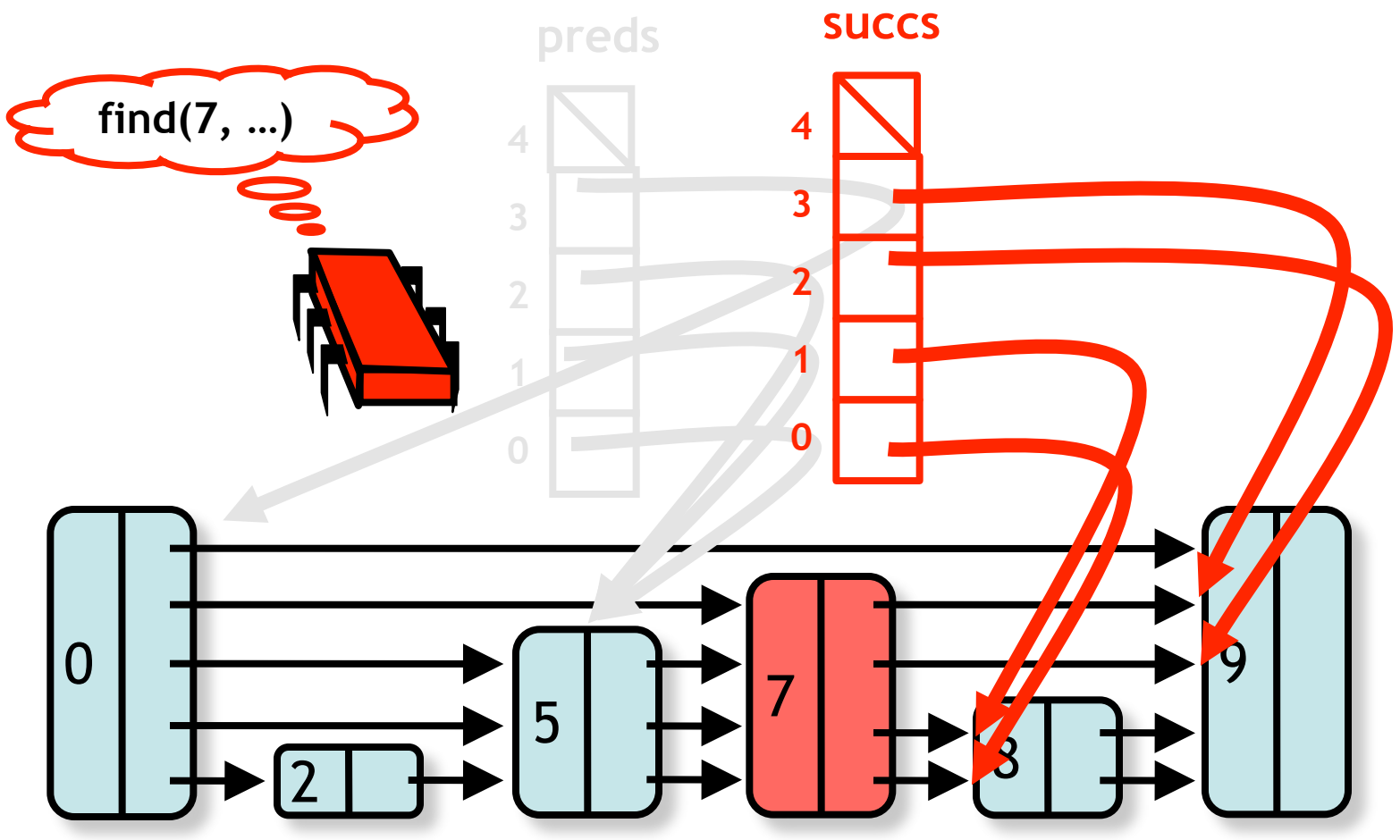
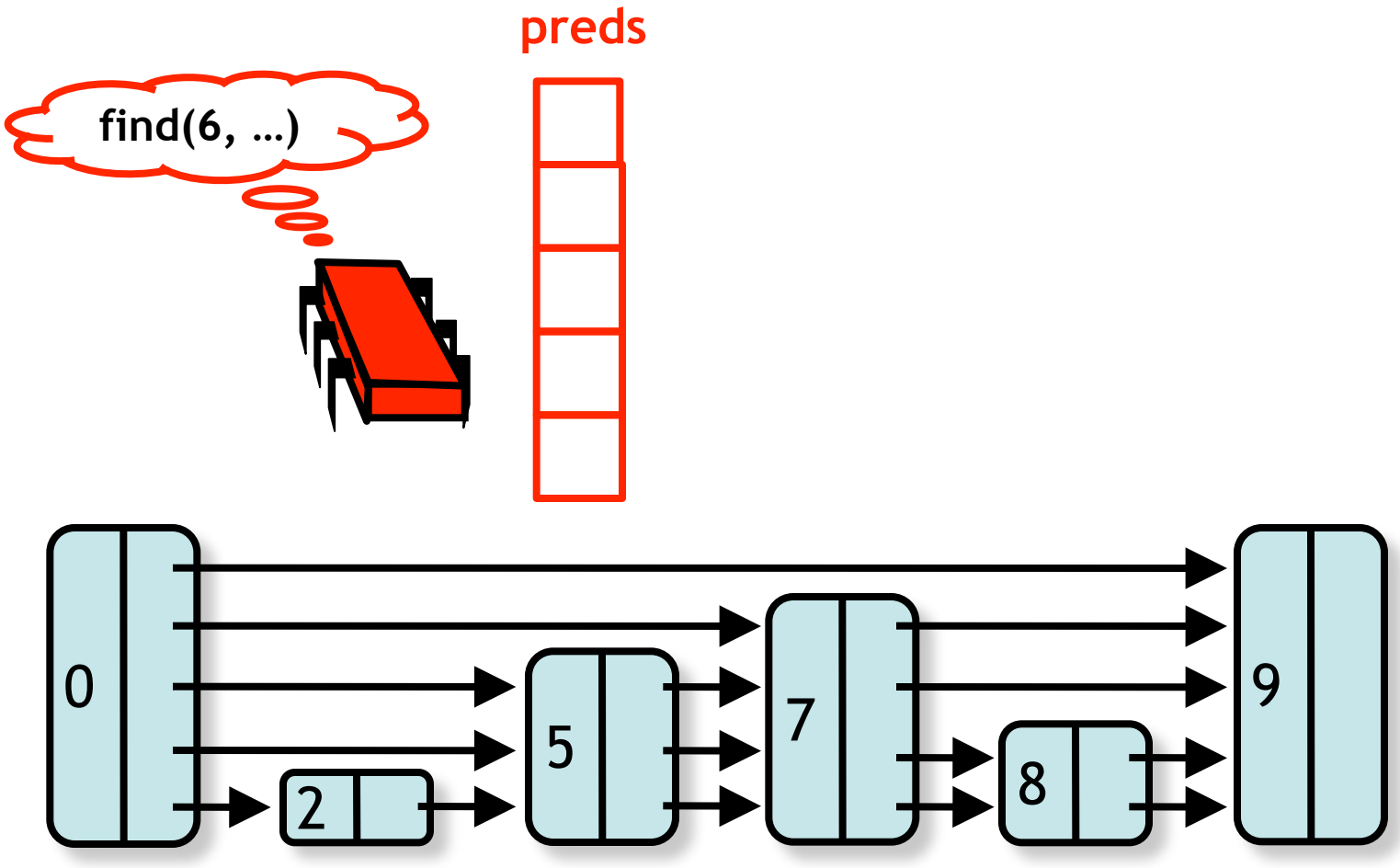


Unsuccessful Search



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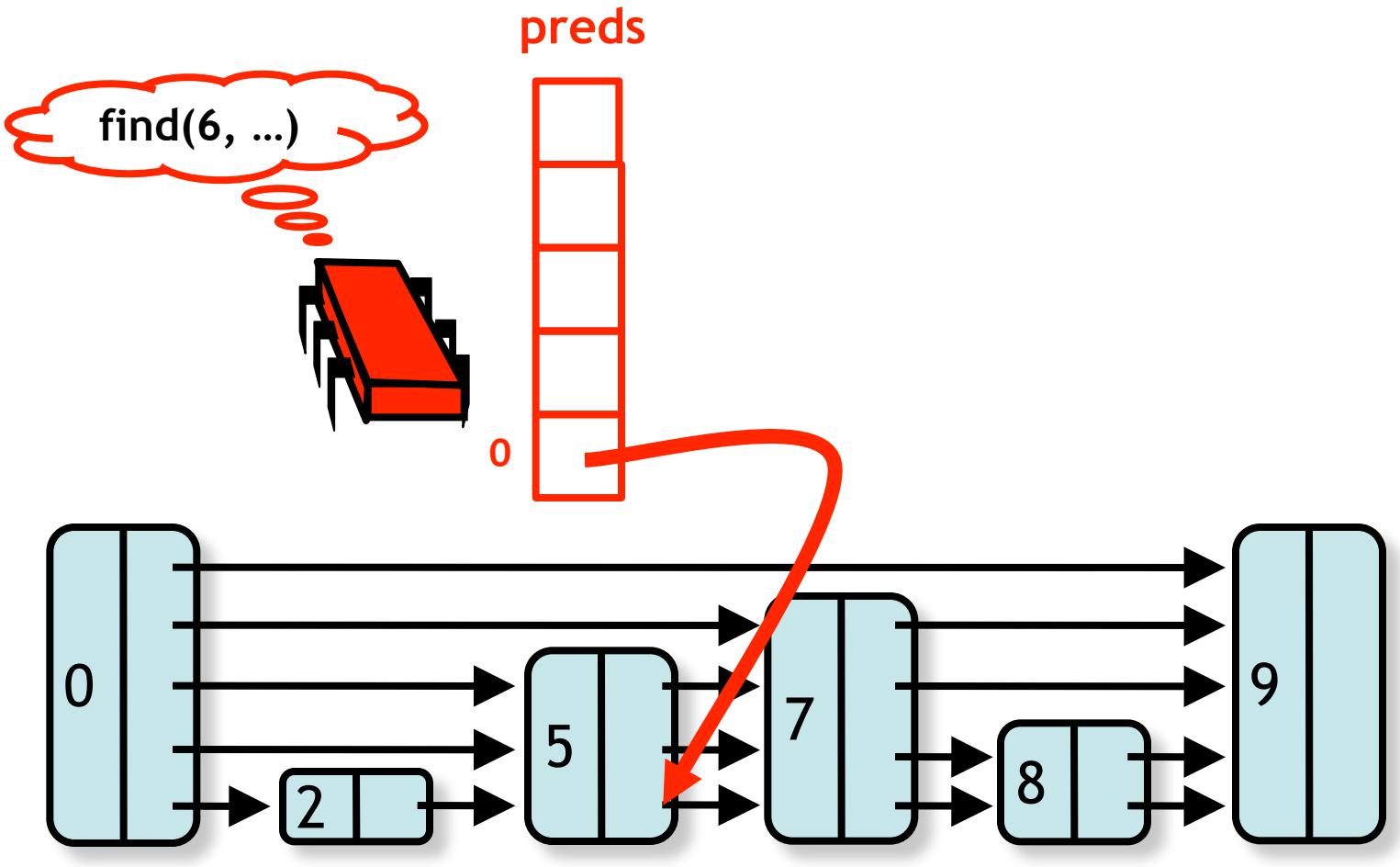

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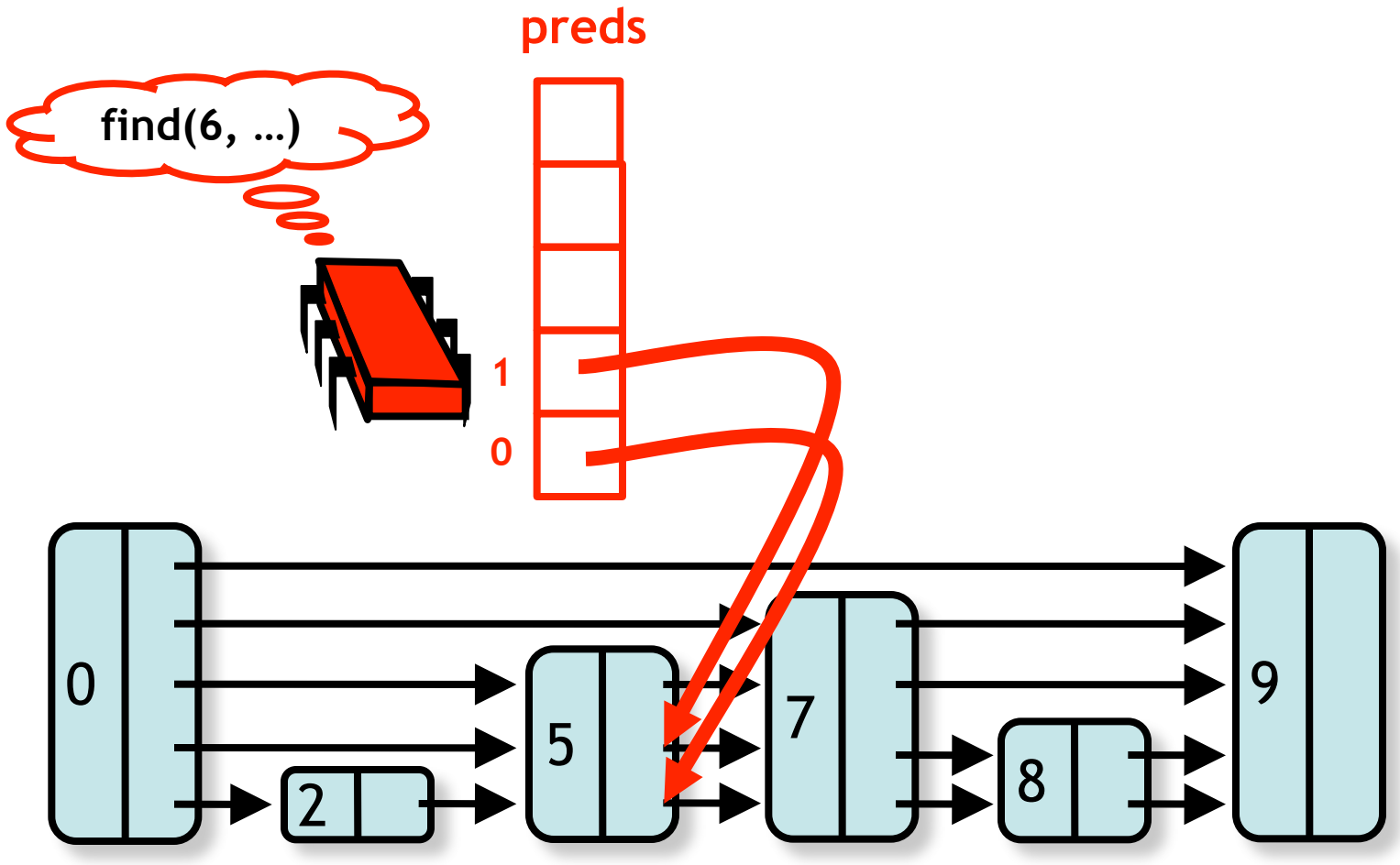

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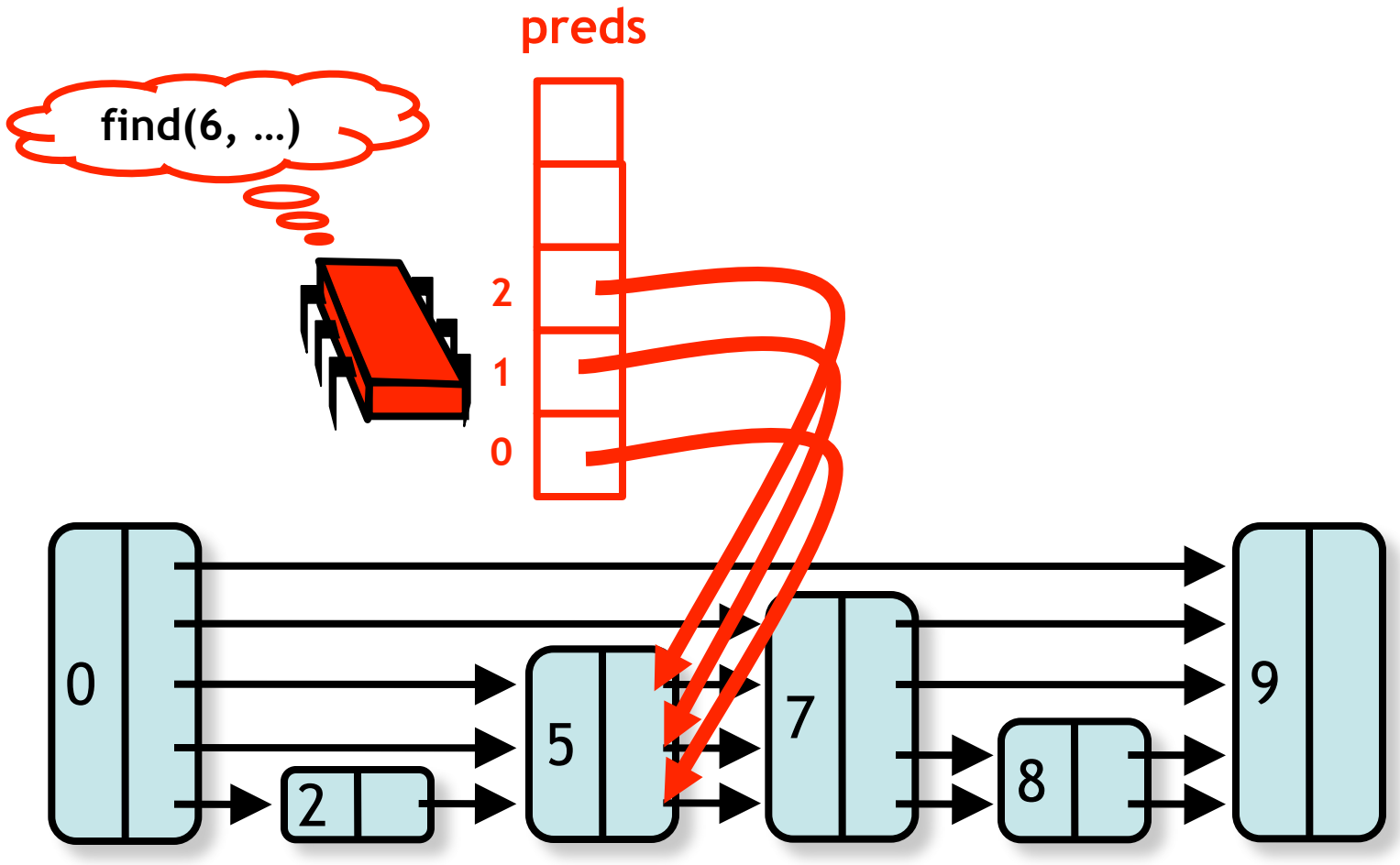

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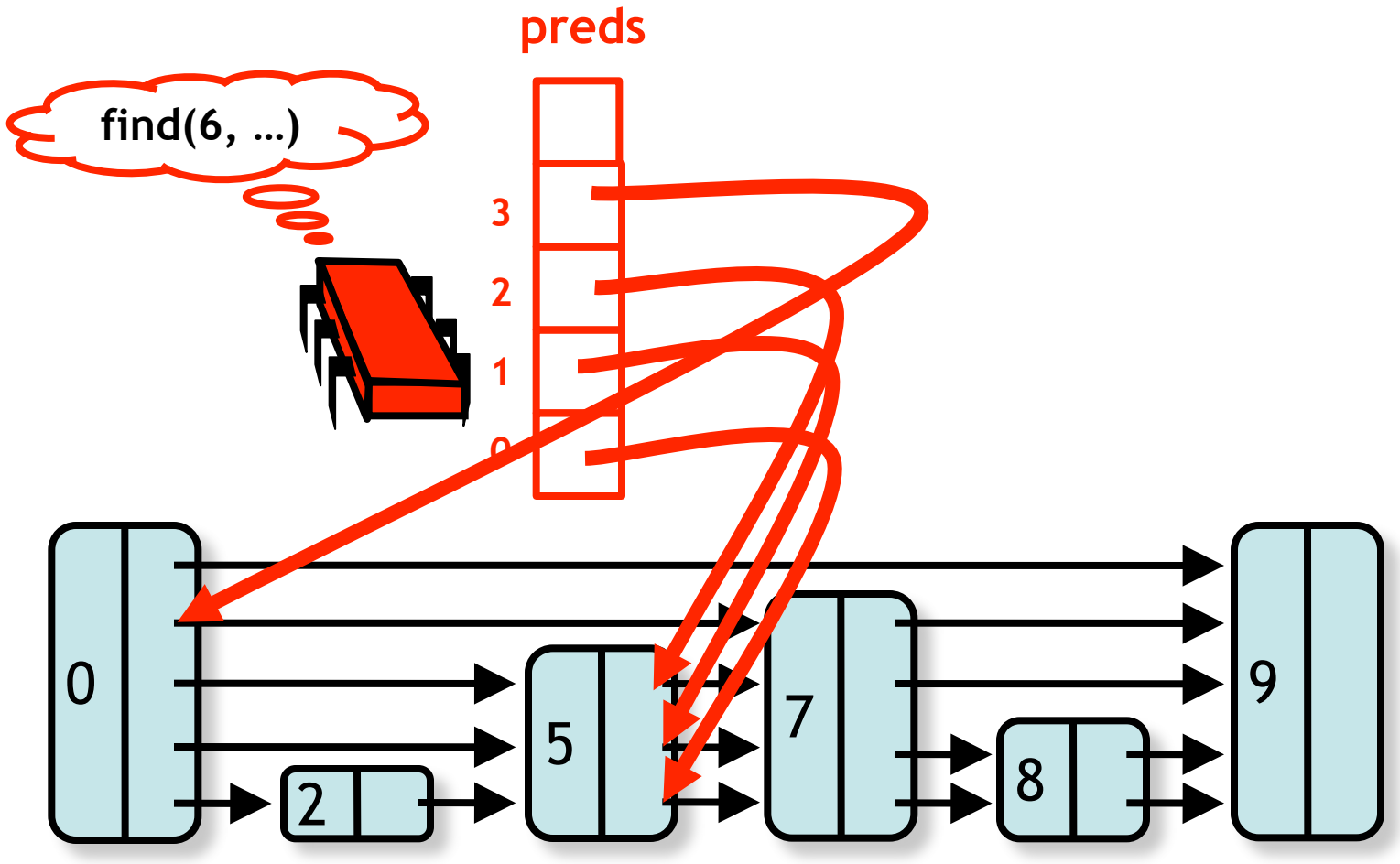

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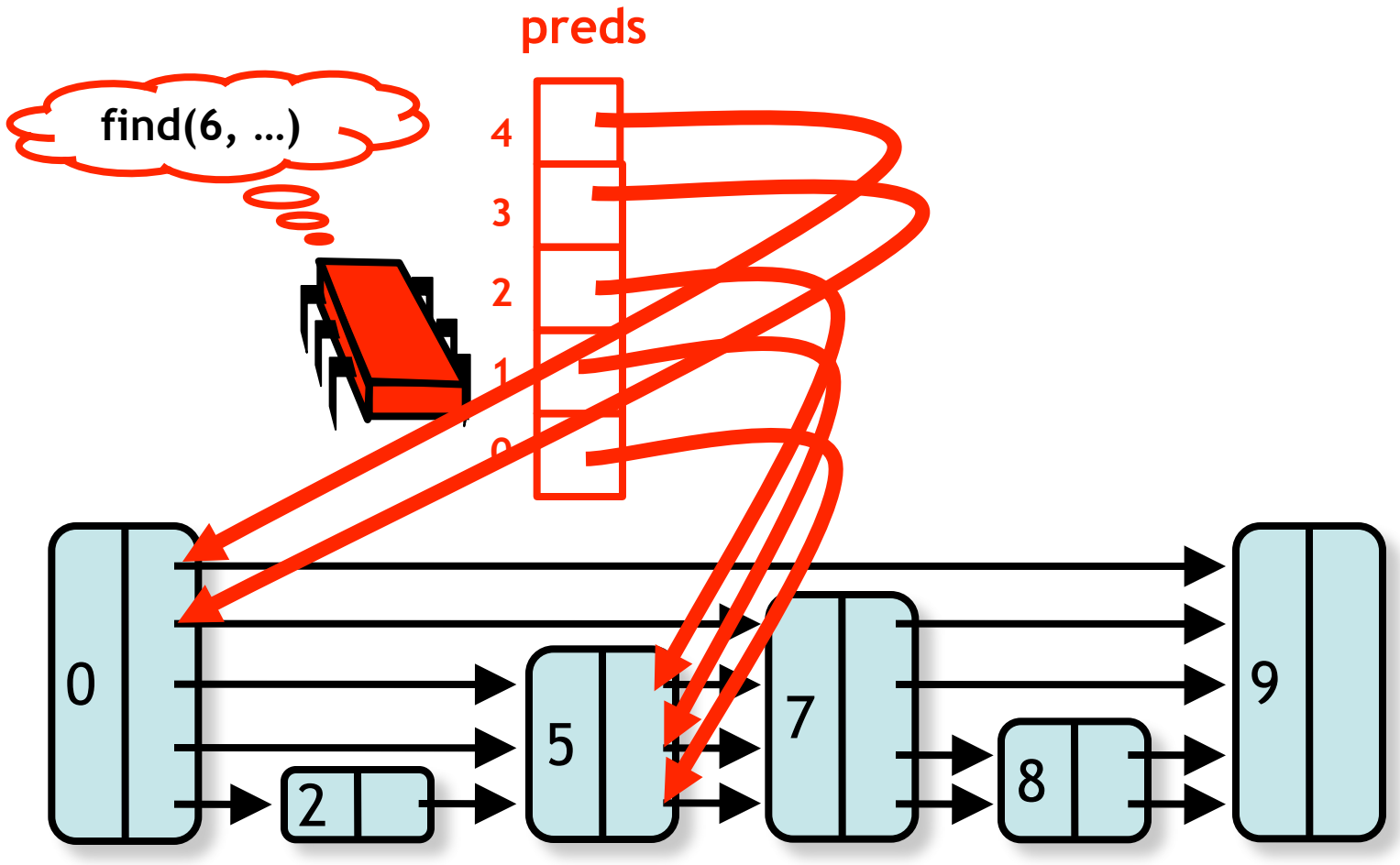

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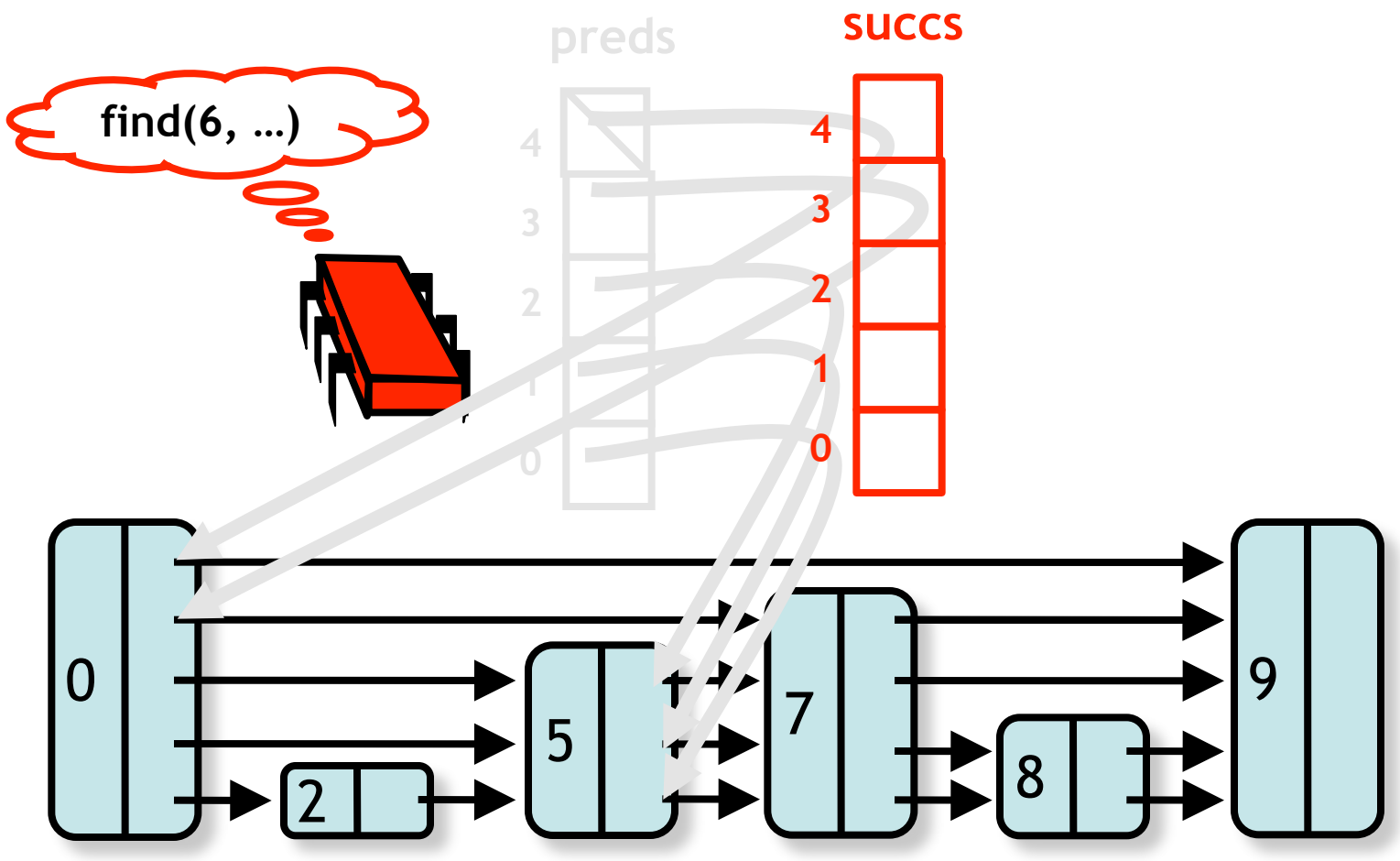

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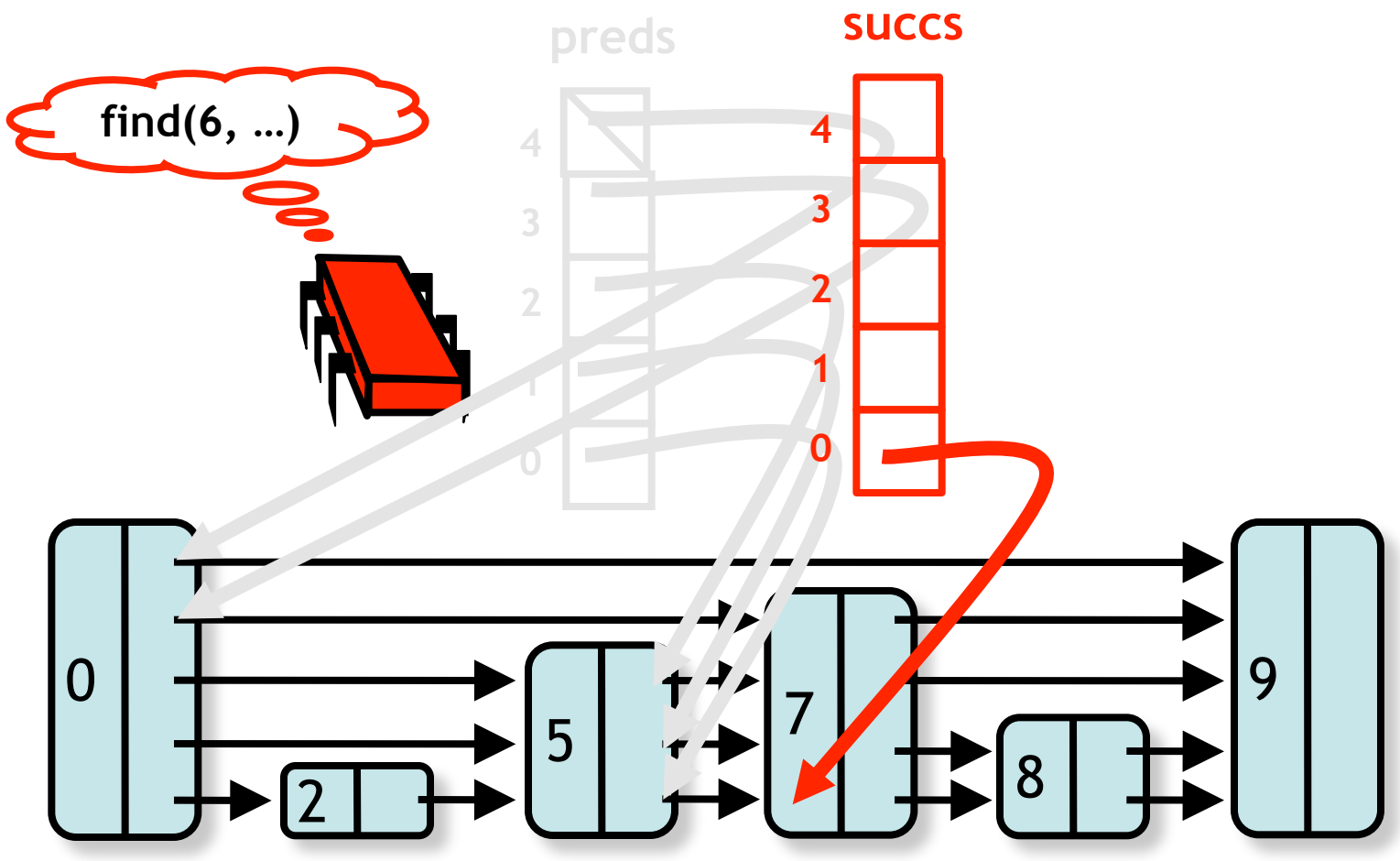

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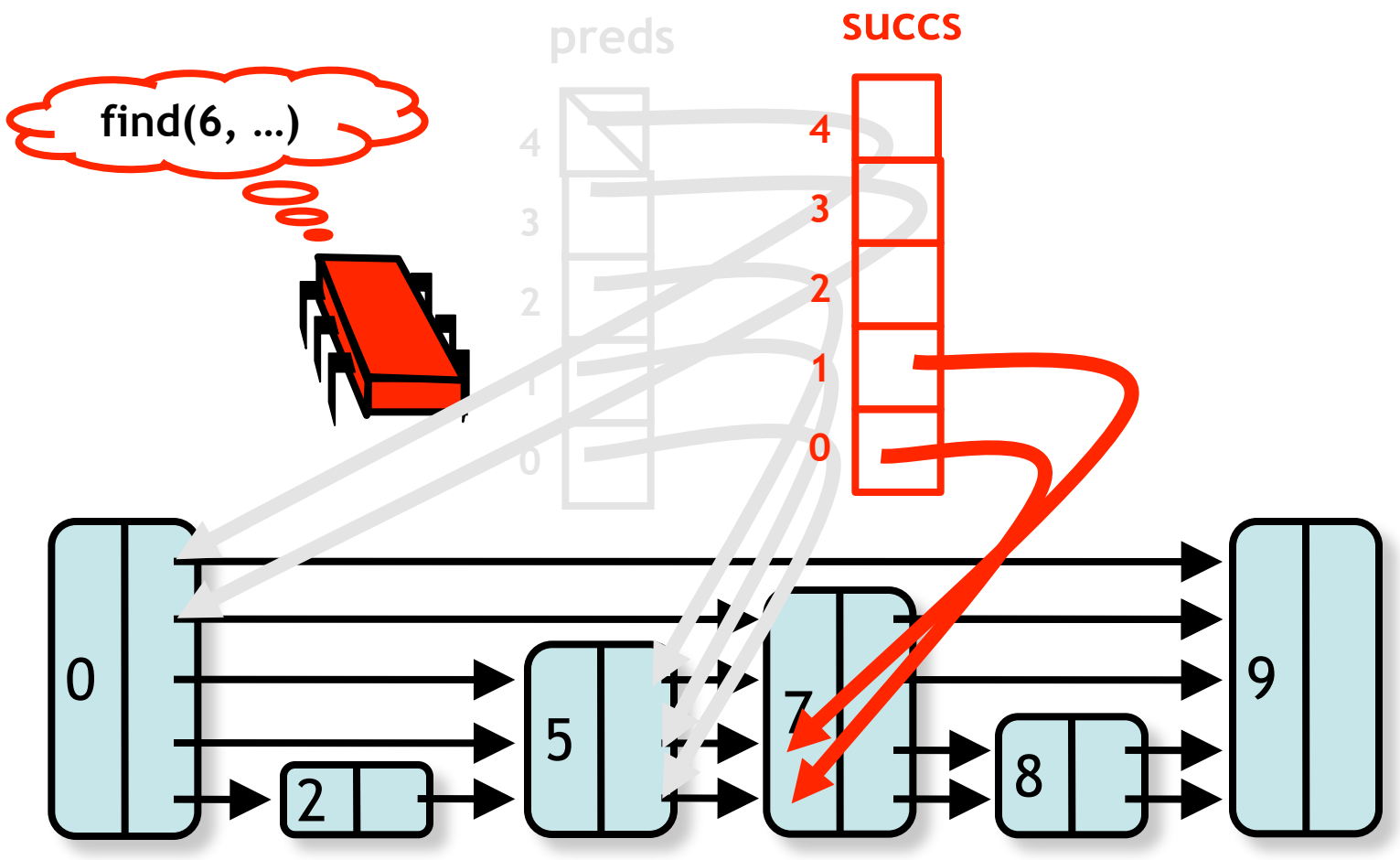

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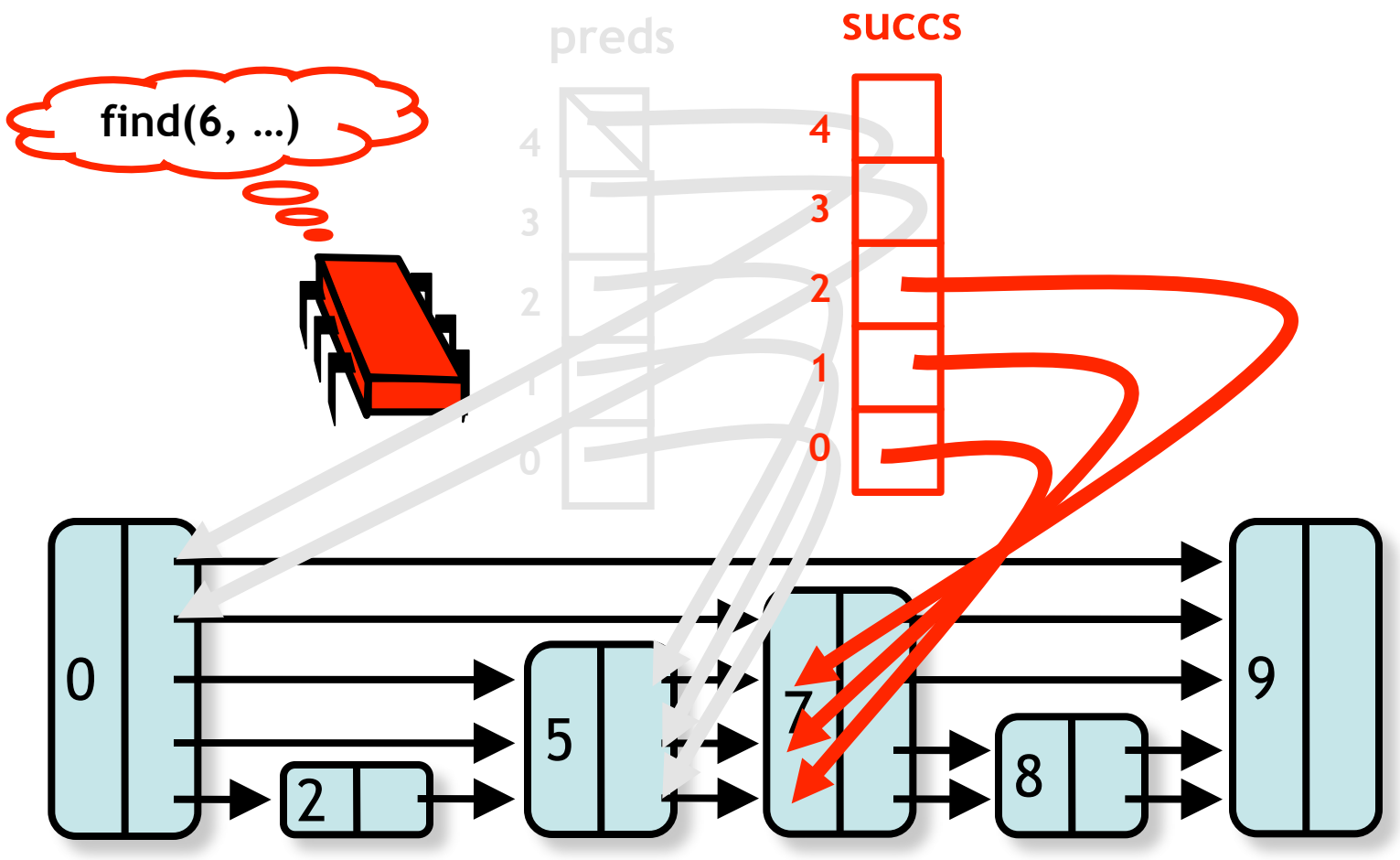

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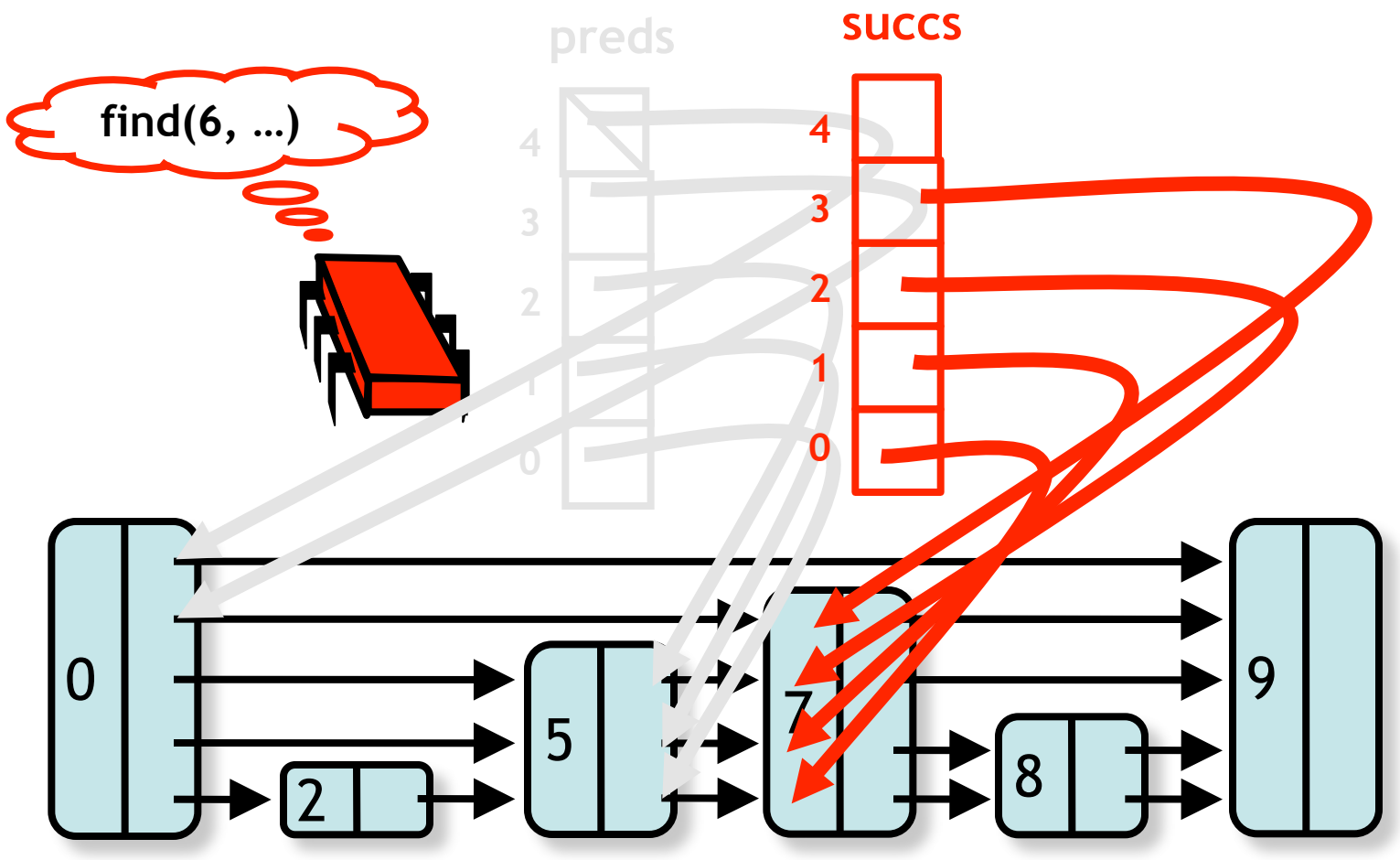

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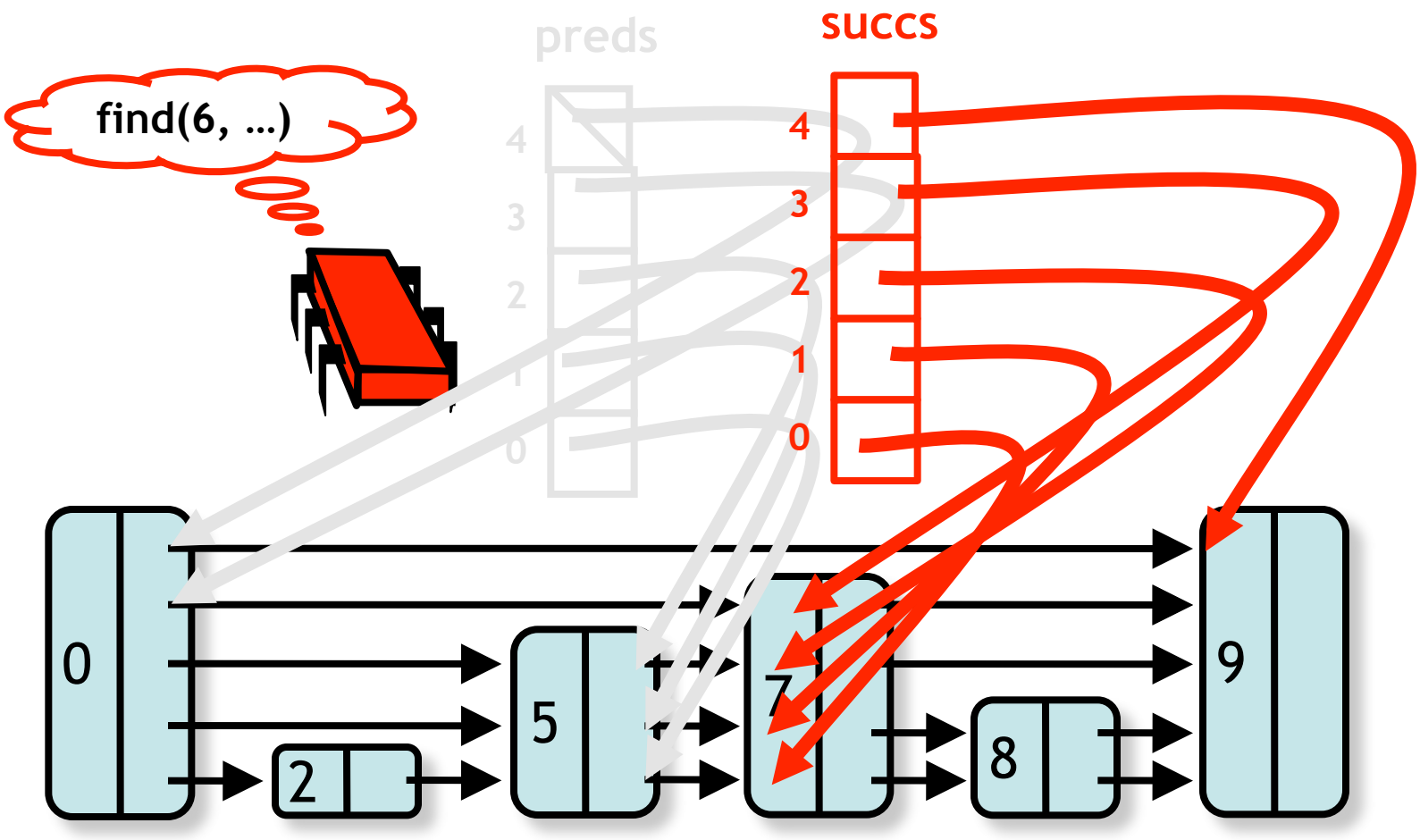
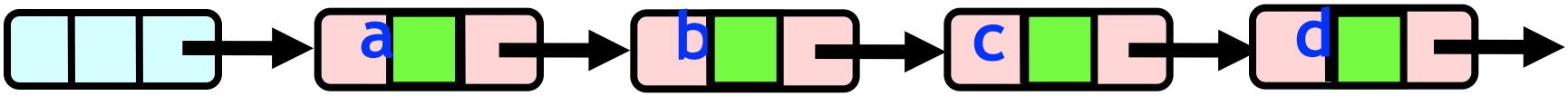


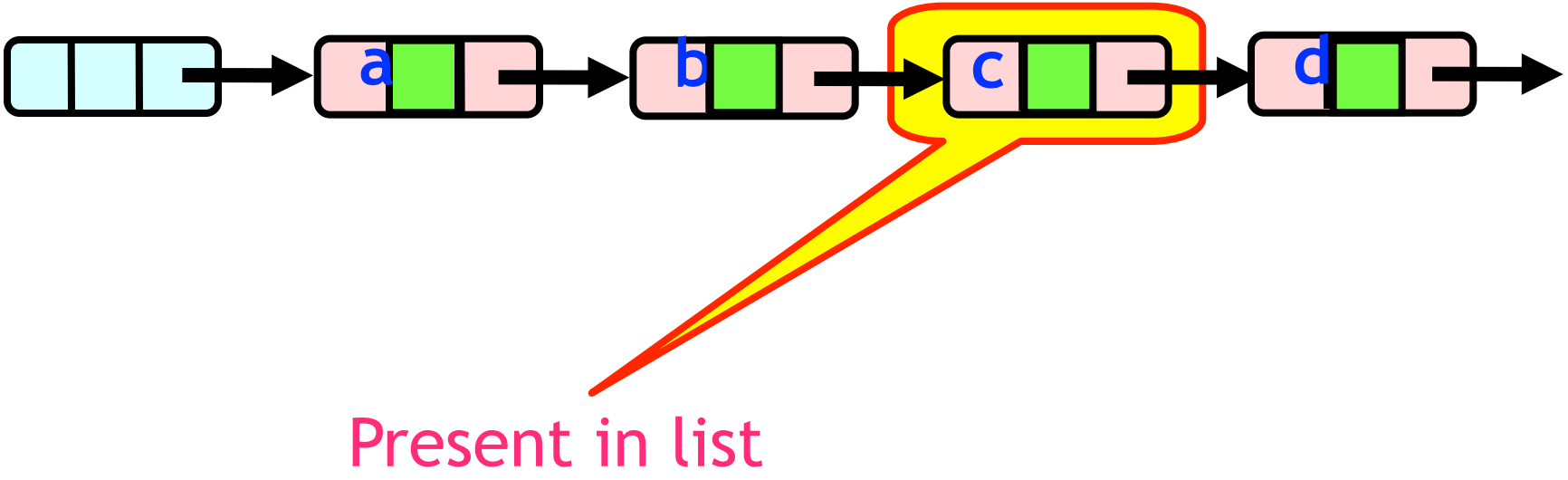
Lazy Skip List

- Mix blocking and non-blocking techniques:
  - Use optimistic-lazy locking for add() and remove()
  - Wait-free contains()
- Remember: typically lots of contains() calls but few add() and remove()

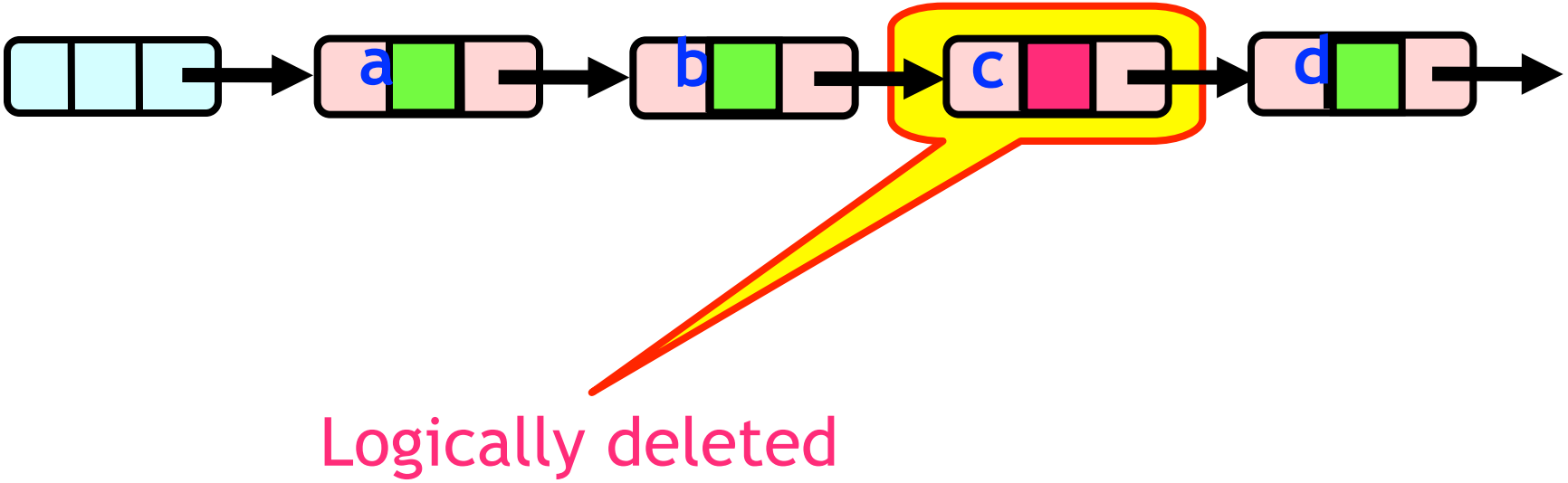
Review: Lazy List Remove



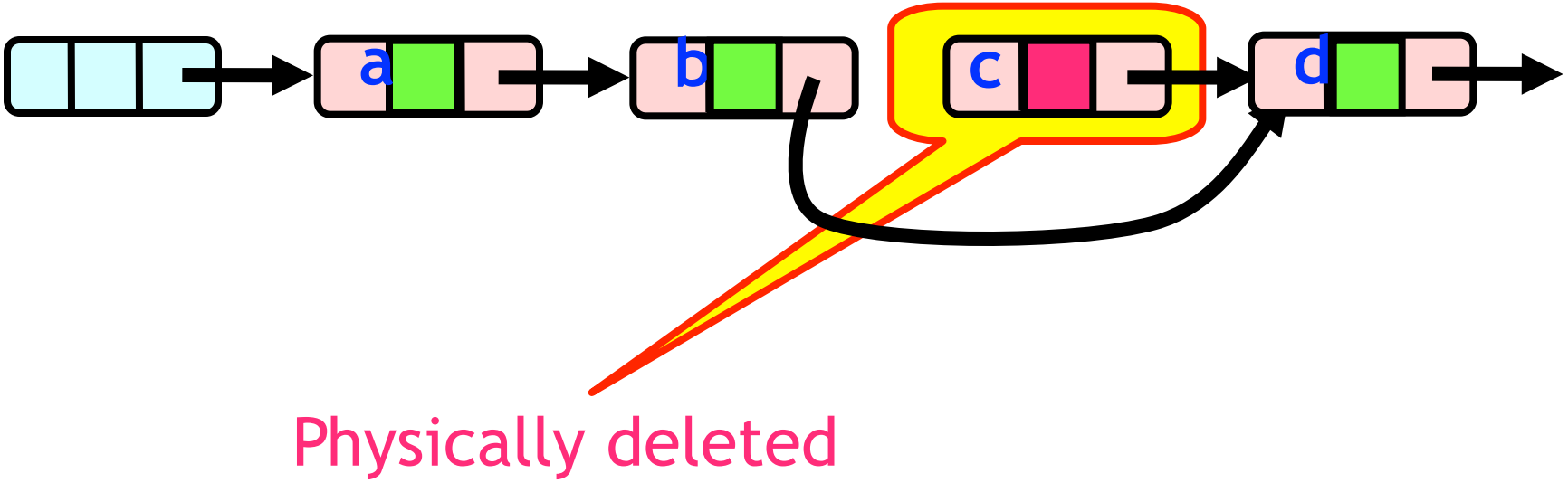
Review: Lazy List Remove



Review: Lazy List Remove

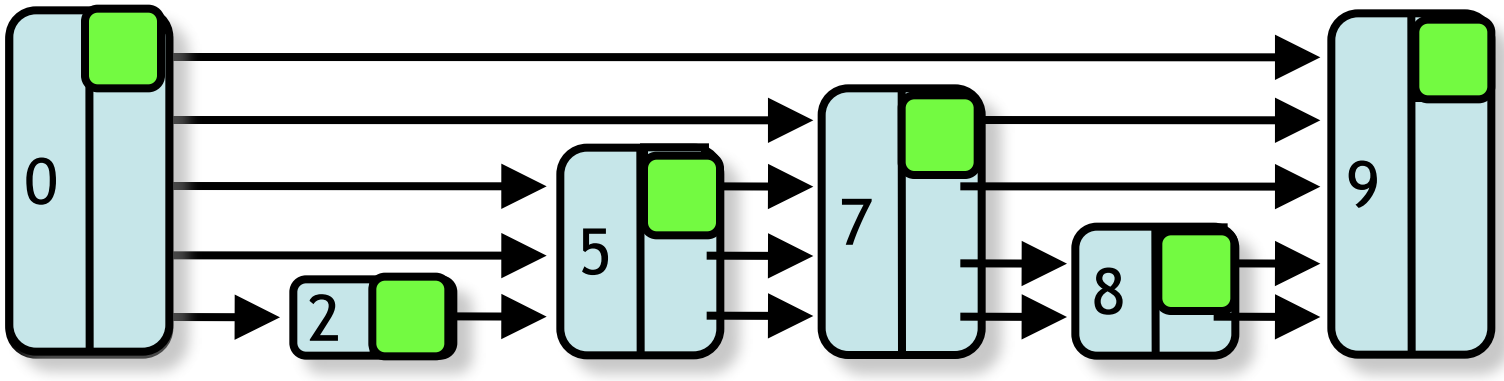


Review: Lazy List Remove



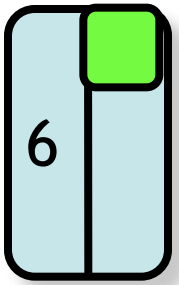
Lazy Skip Lists

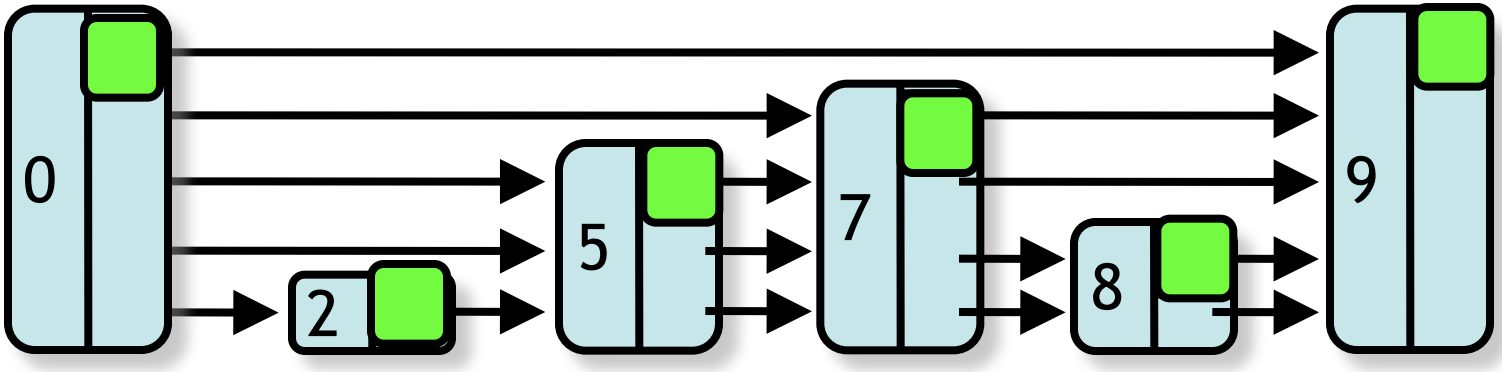
- Use a mark bit for logical deletion



add(6)

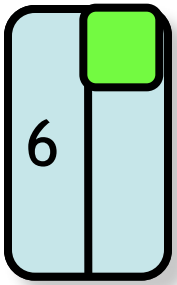
- Create node of (random) height 4

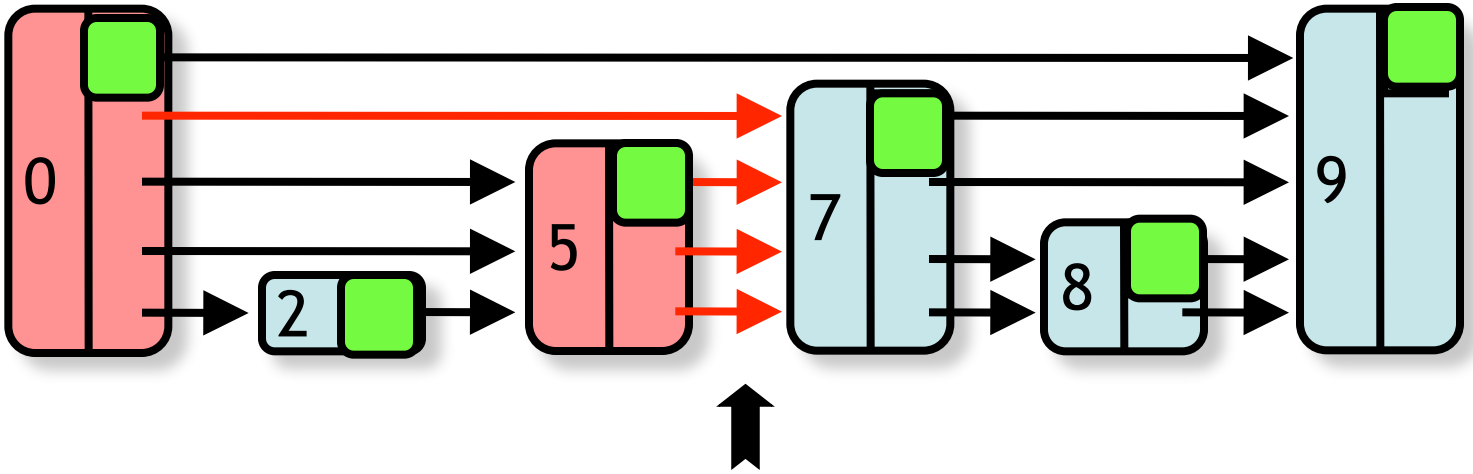




add(6)

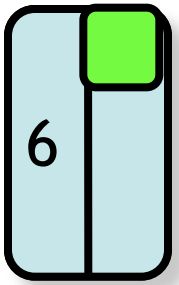
- **find()** predecessors

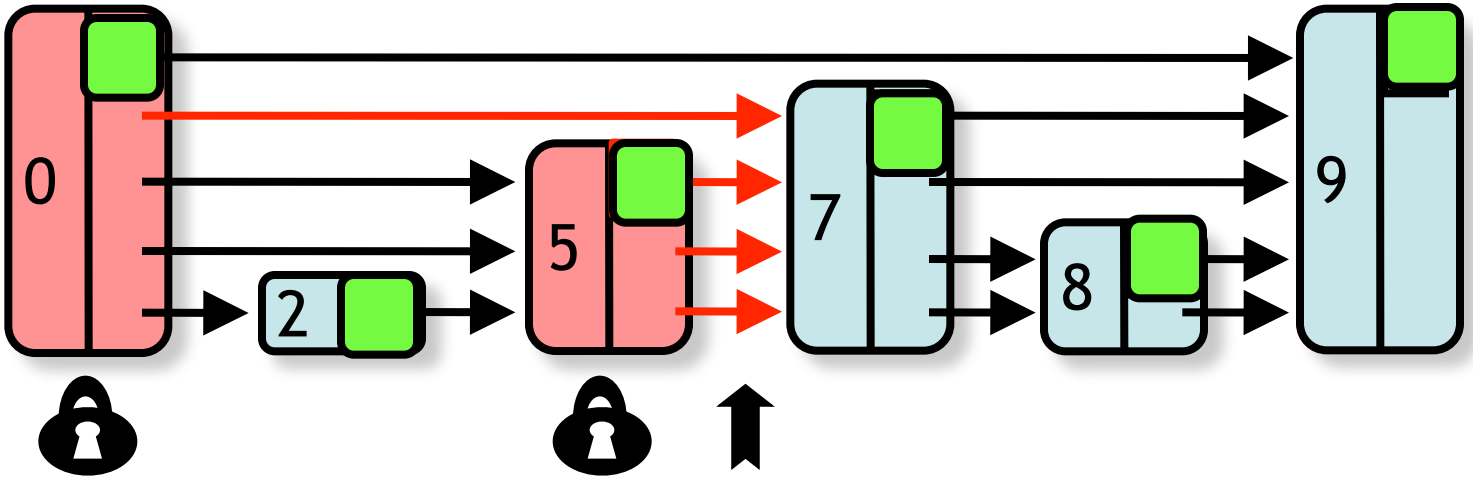




add(6)

- **find()** predecessors
- Lock them

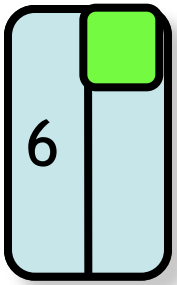


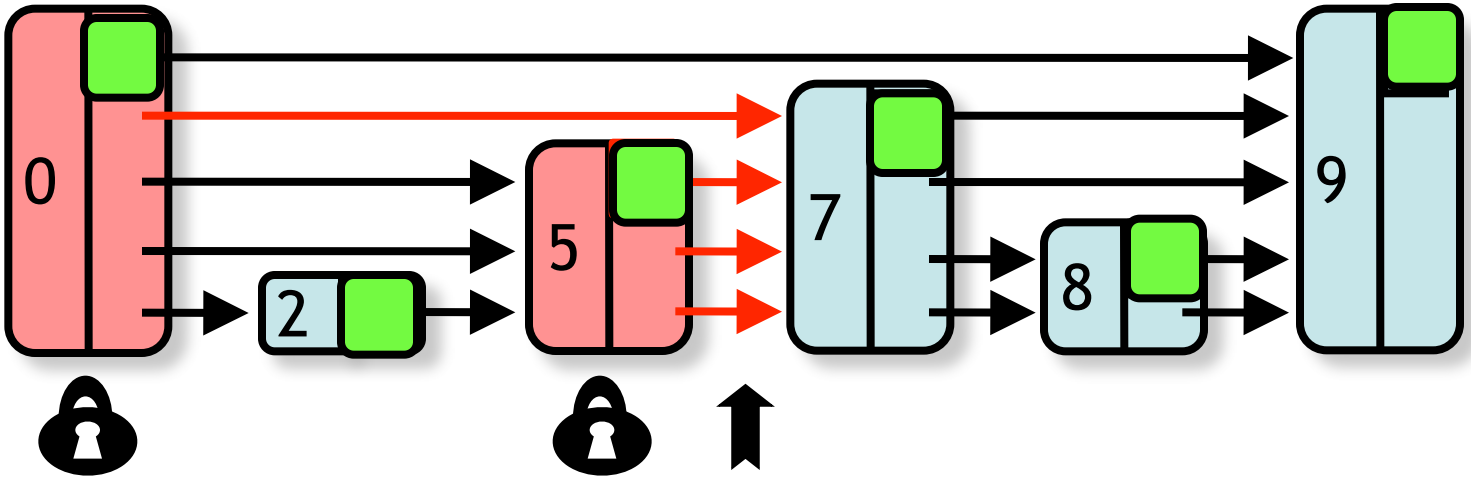


add(6)

- **find()** predecessors
- Lock them
- Validate

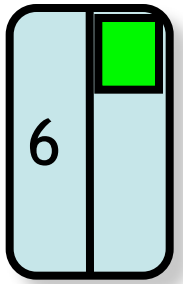
} Optimistic approach

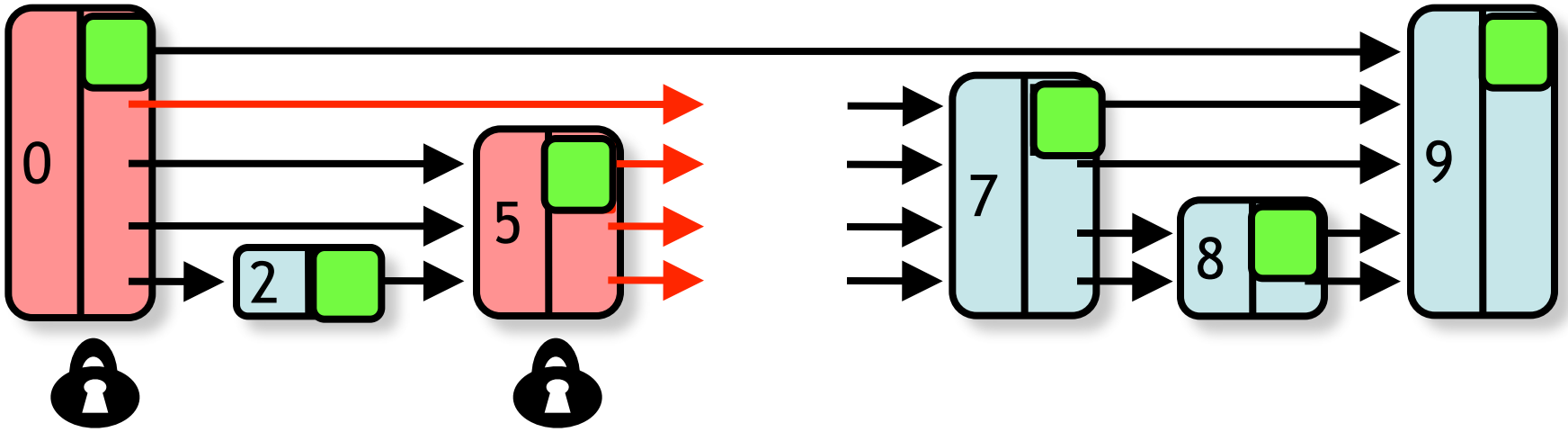




add(6)

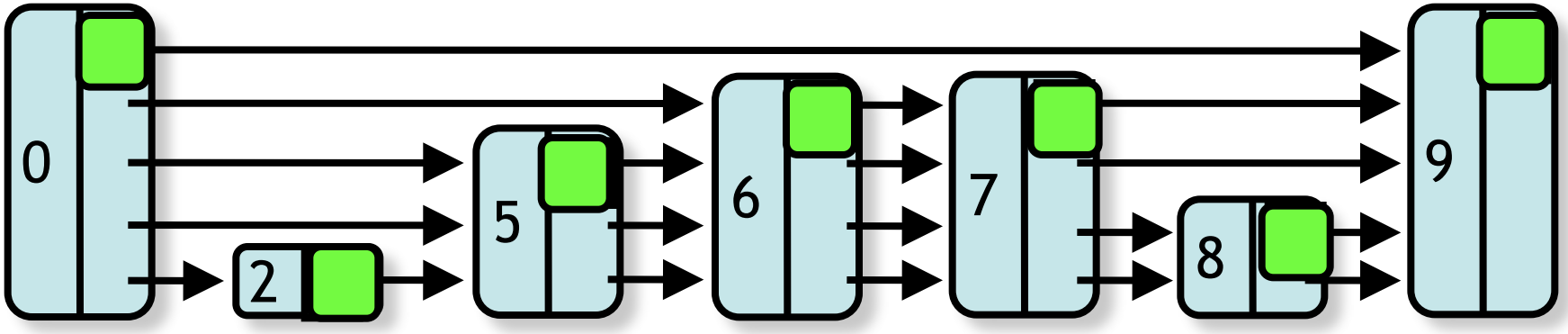
- **find()** predecessors
- Lock them
- Validate
- Splice



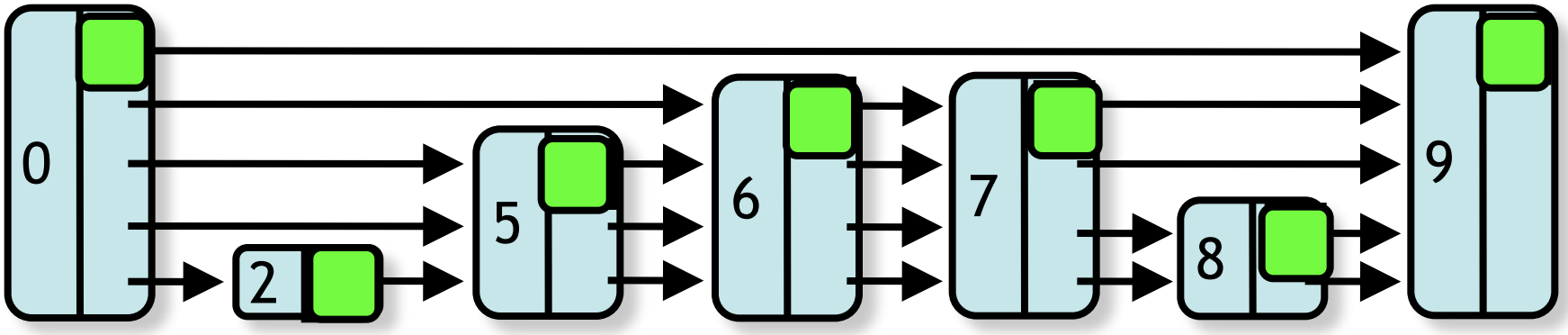


add(6)

- **find()** predecessors
- Lock them
- Validate
- Splice
- Unlock

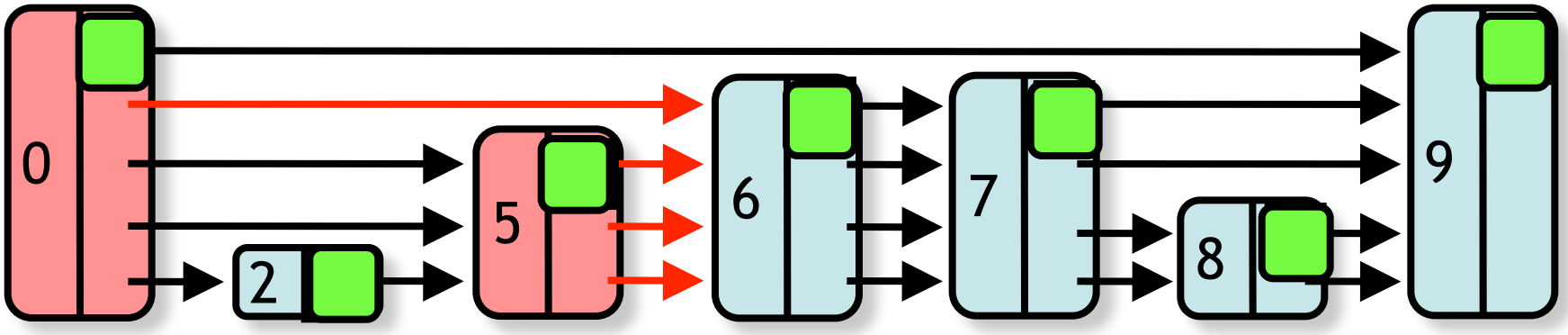


remove(6)



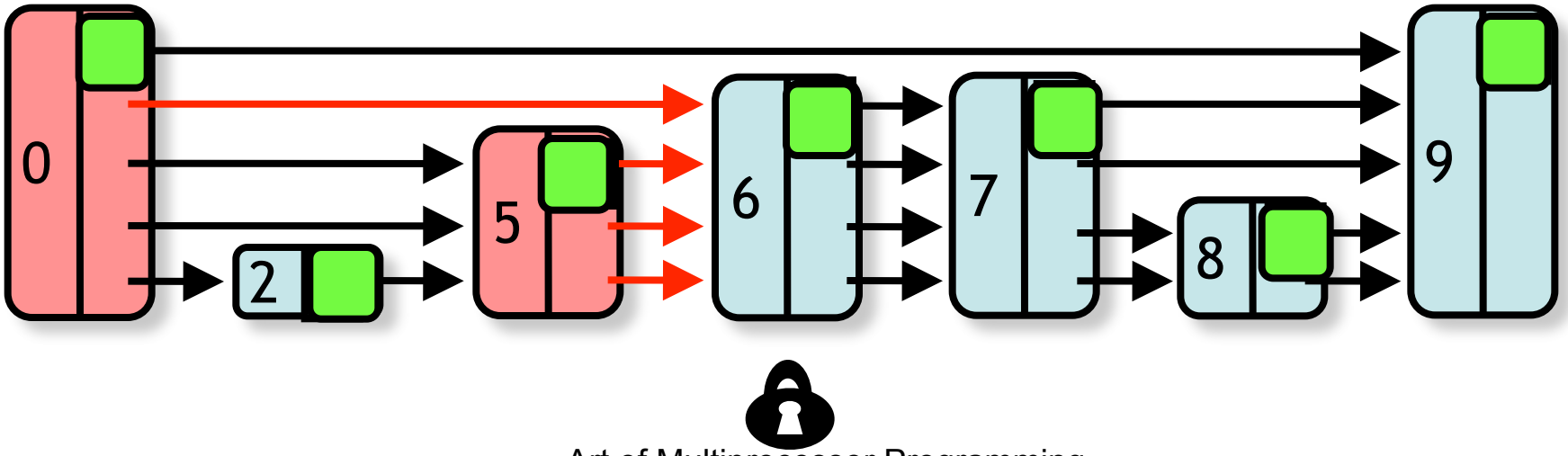
remove(6)

- **find()** predecessors



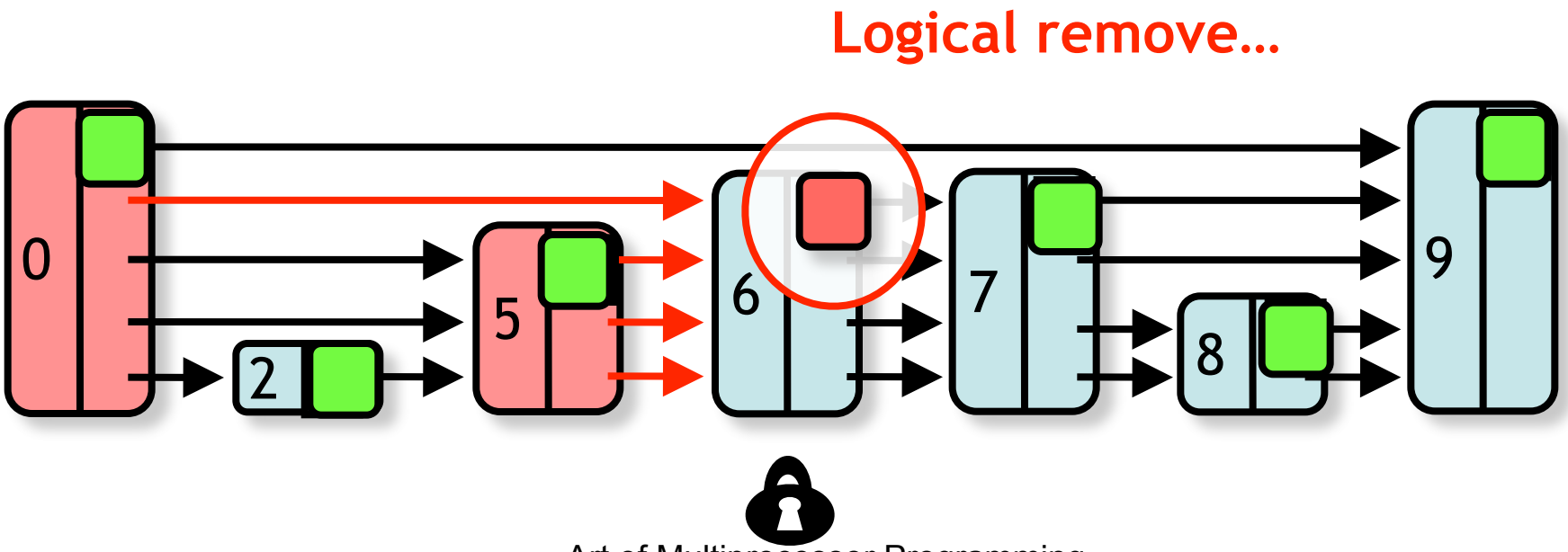
remove(6)

- **find()** predecessors
- Lock victim



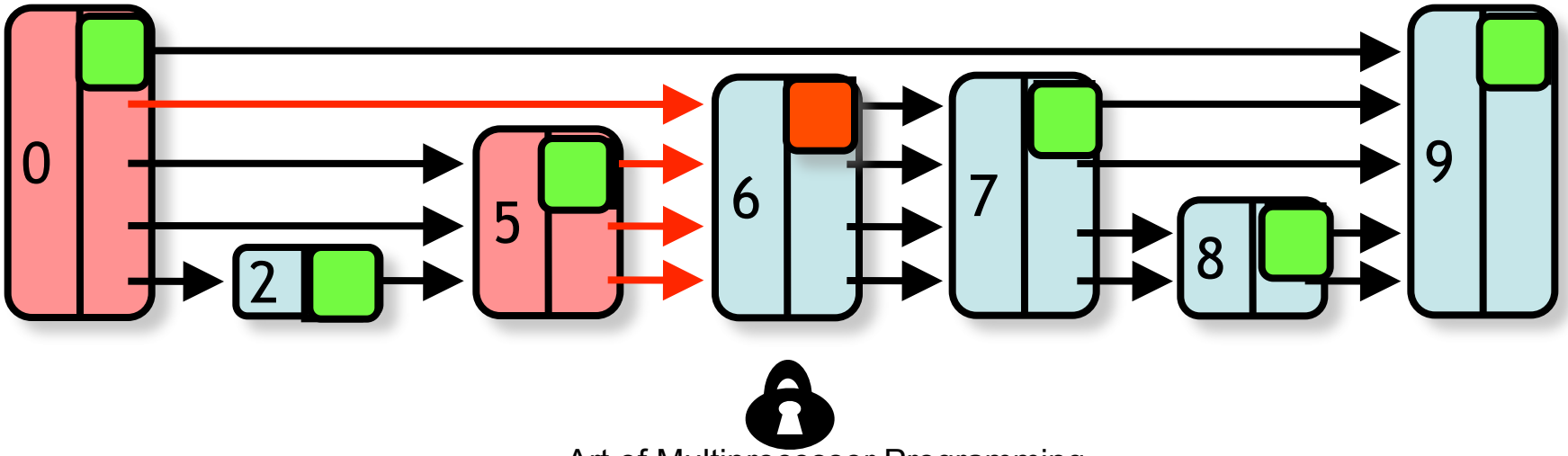
remove(6)

- **find()** predecessors
- Lock victim
- Set mark (if not already set)



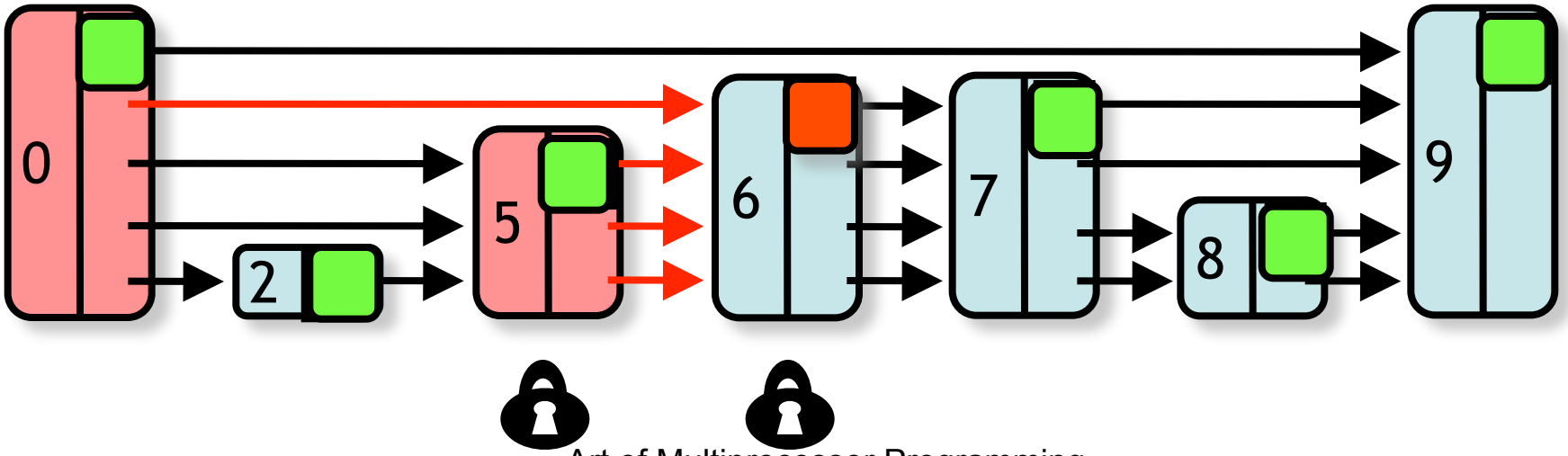
remove(6)

- **find()** predecessors
- Lock victim
- Set mark (if not already set)
- Lock predecessors (ascending order) & validate



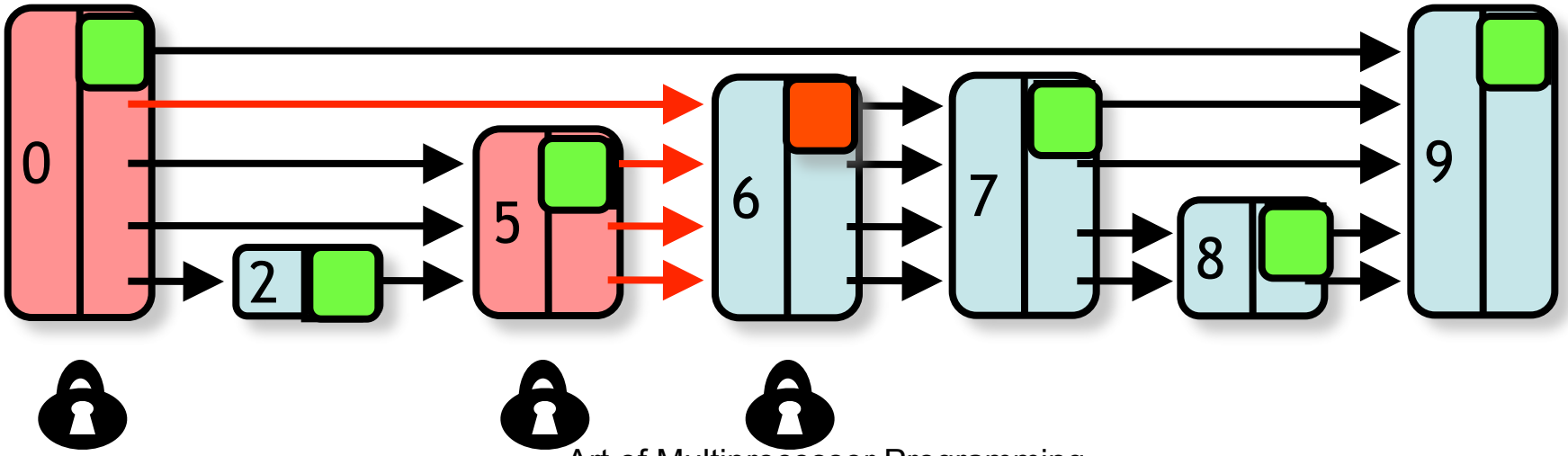
remove(6)

- **find()** predecessors
- Lock victim
- Set mark (if not already set)
- Lock predecessors (ascending order) & validate



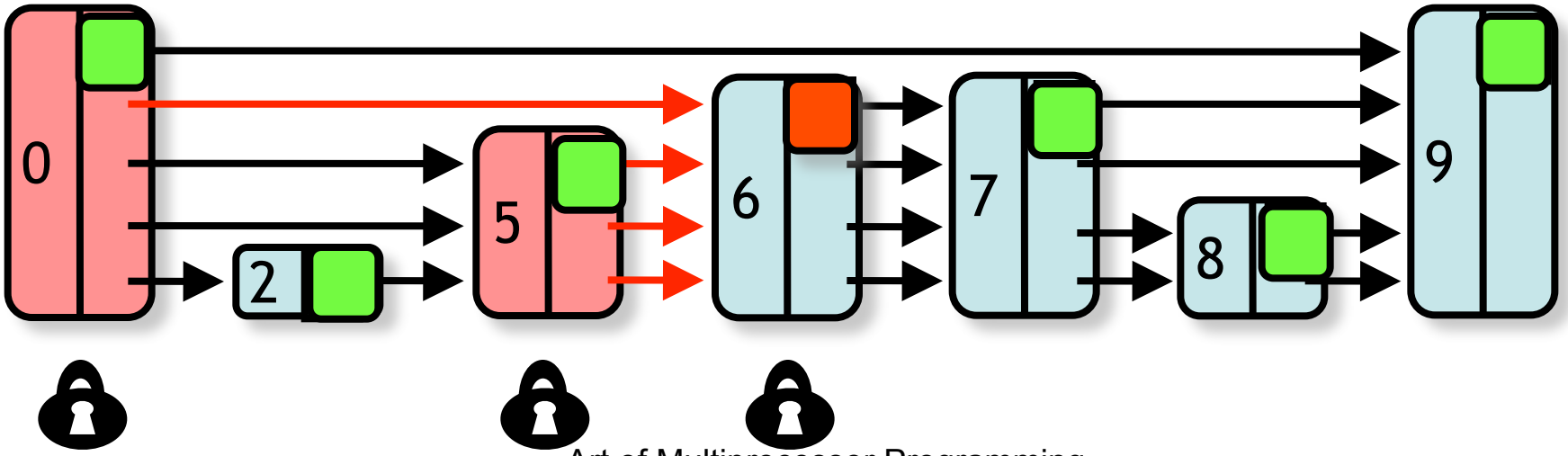
remove(6)

- **find()** predecessors
- Lock victim
- Set mark (if not already set)
- Lock predecessors (ascending order) & validate



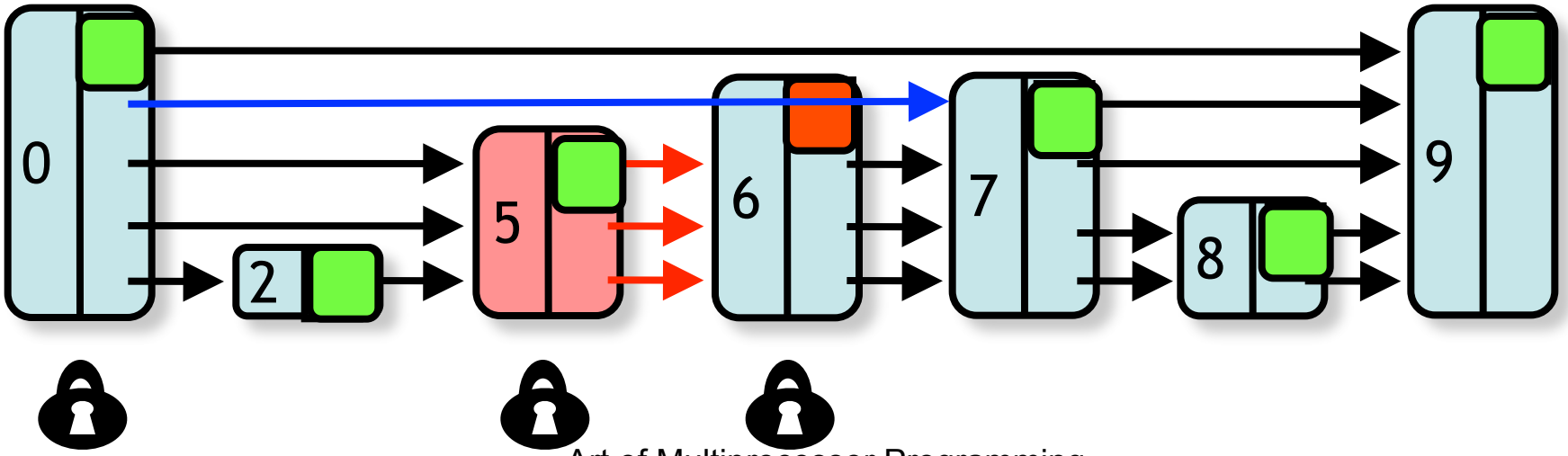
remove(6)

- **find()** predecessors
- Lock victim
- Set mark (if not already set)
- Lock predecessors (ascending order) & validate
- Physically remove



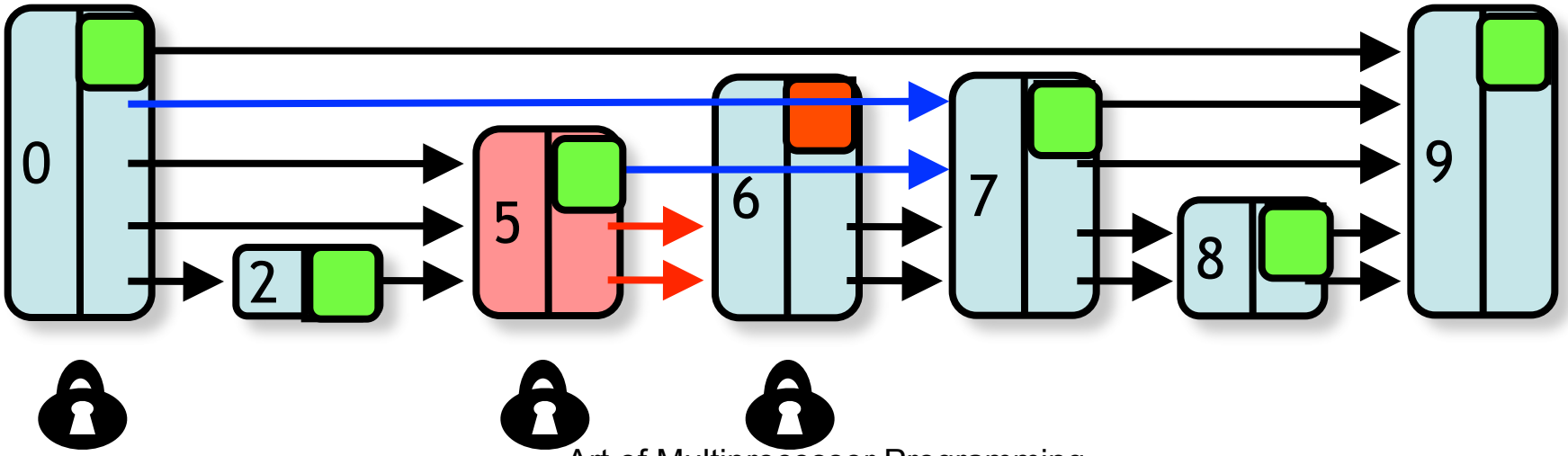
remove(6)

- **find()** predecessors
- Lock victim
- Set mark (if not already set)
- Lock predecessors (ascending order) & validate
- Physically remove



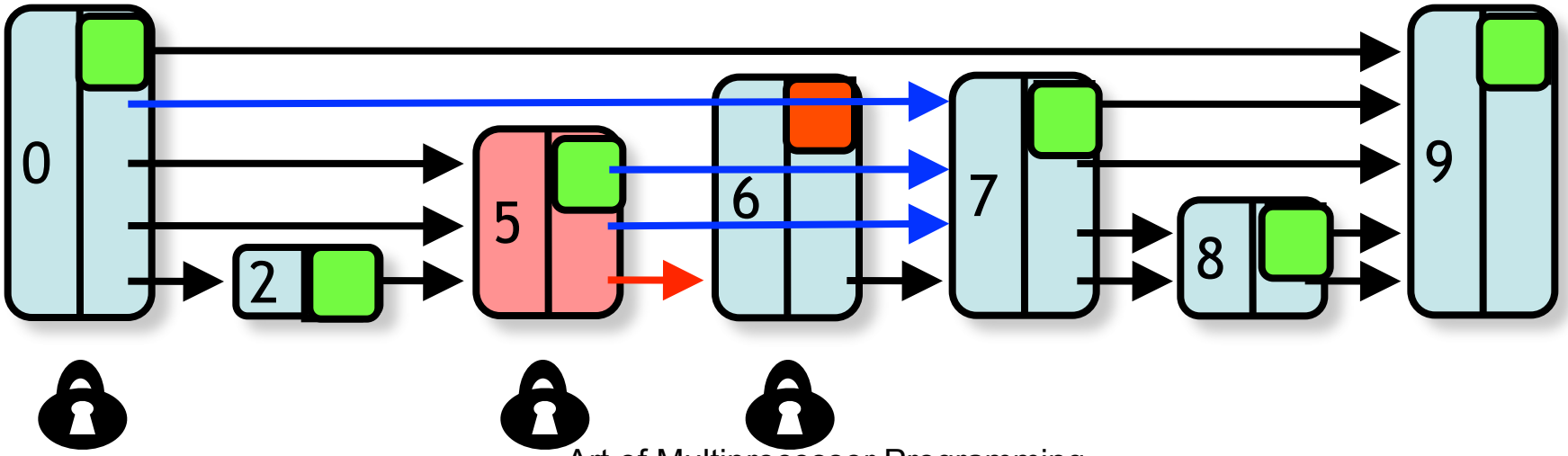
remove(6)

- **find()** predecessors
- Lock victim
- Set mark (if not already set)
- Lock predecessors (ascending order) & validate
- Physically remove



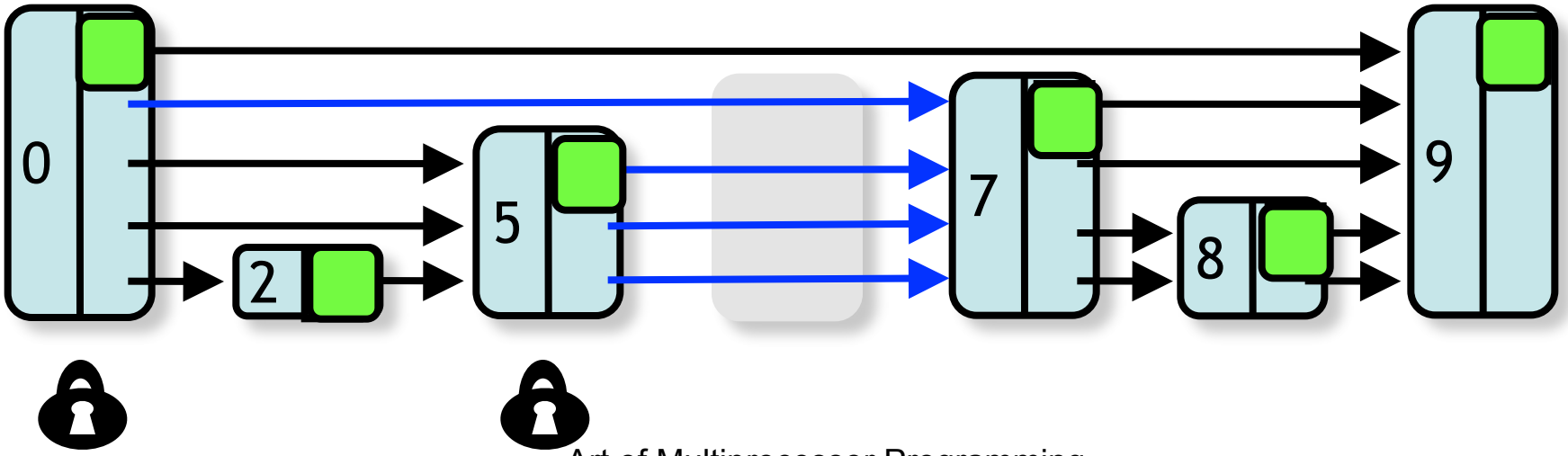
remove(6)

- **find()** predecessors
- Lock victim
- Set mark (if not already set)
- Lock predecessors (ascending order) & validate
- Physically remove



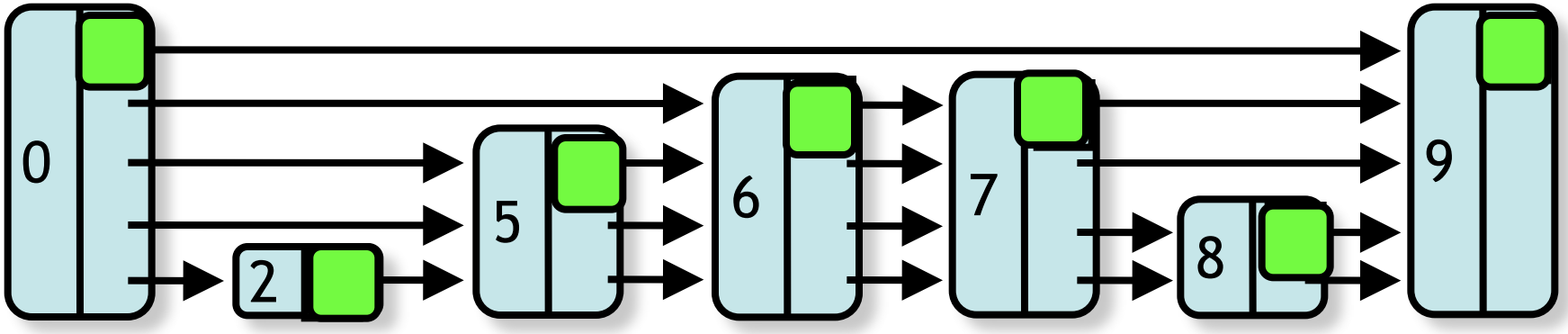
remove(6)

- **find()** predecessors
- Lock victim
- Set mark (if not already set)
- Lock predecessors (ascending order) & validate
- Physically remove

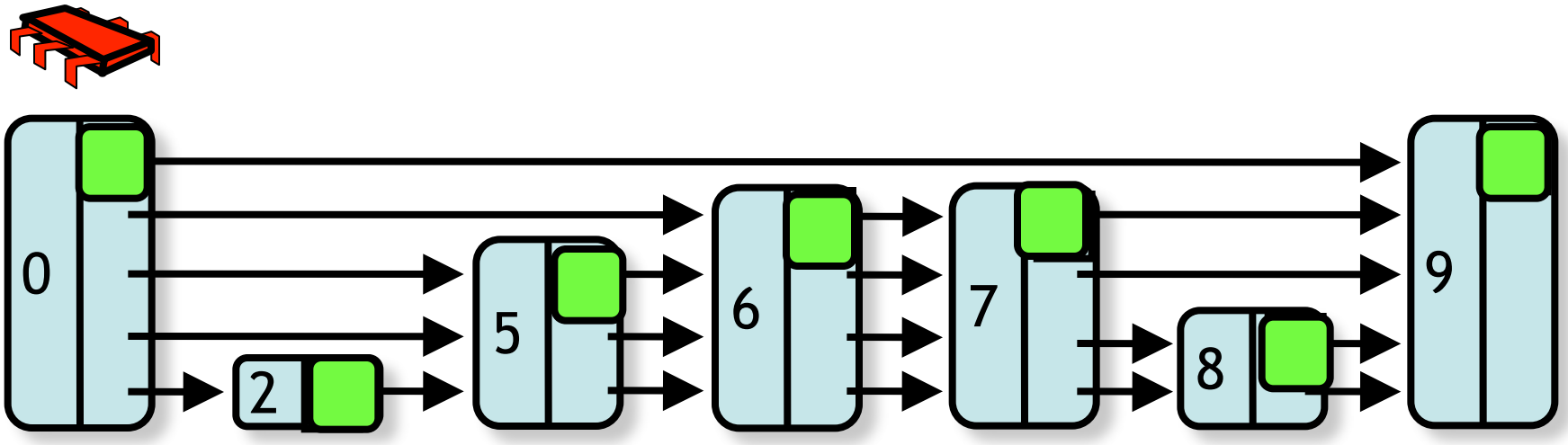


contains(8)

- Find() & not marked

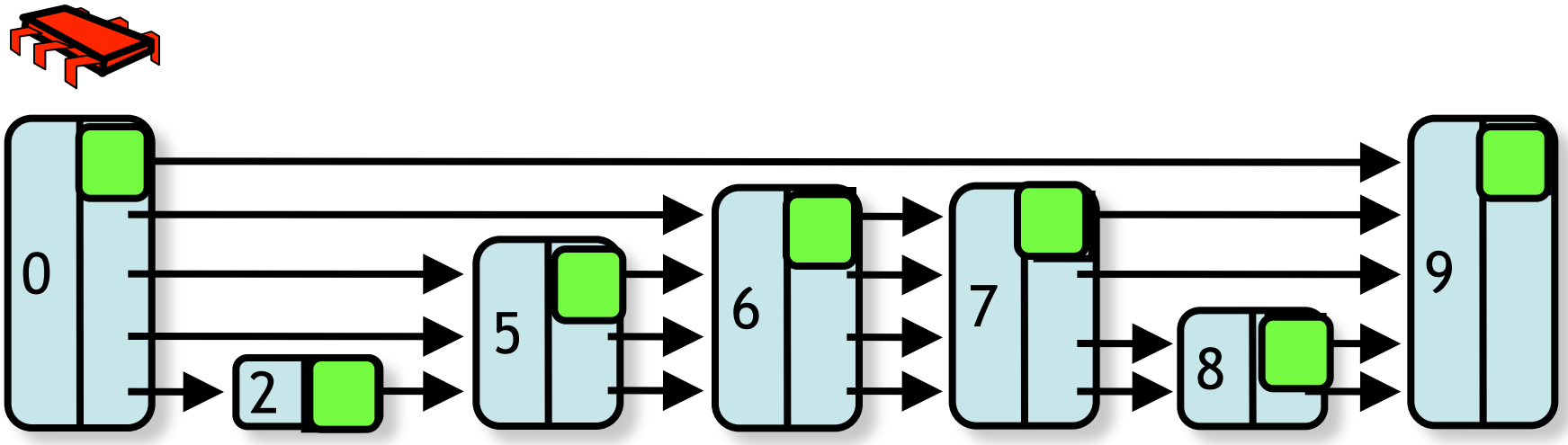


contains(8)



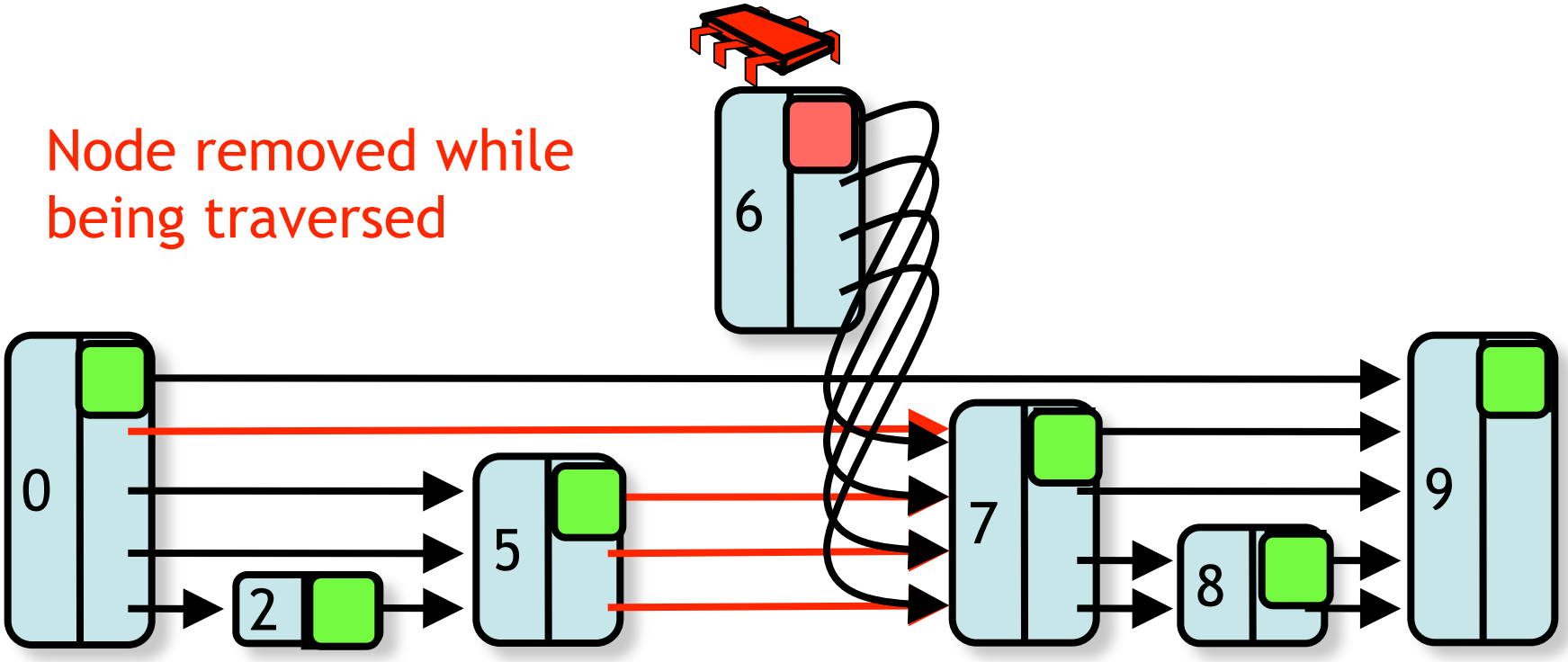
contains(8)

Node 6 removed while traversed



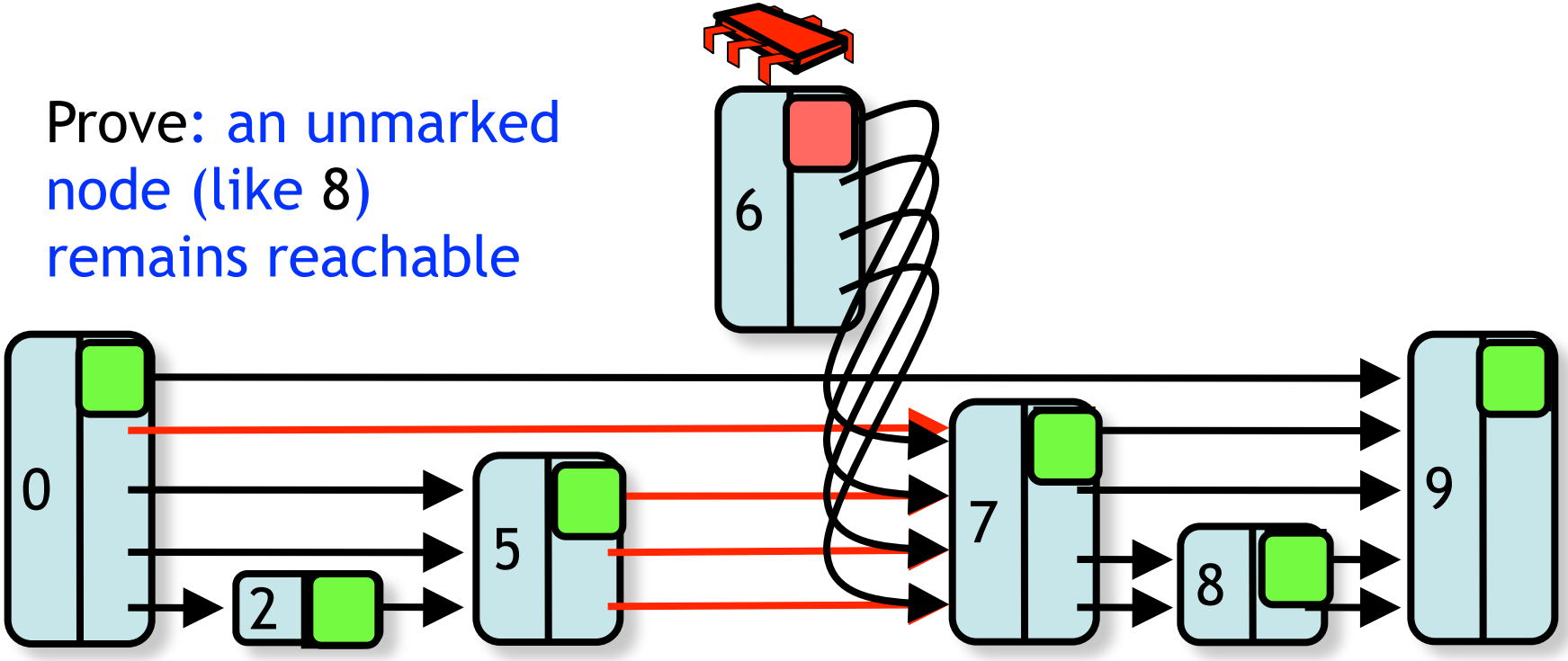
contains(8)

Node removed while being traversed



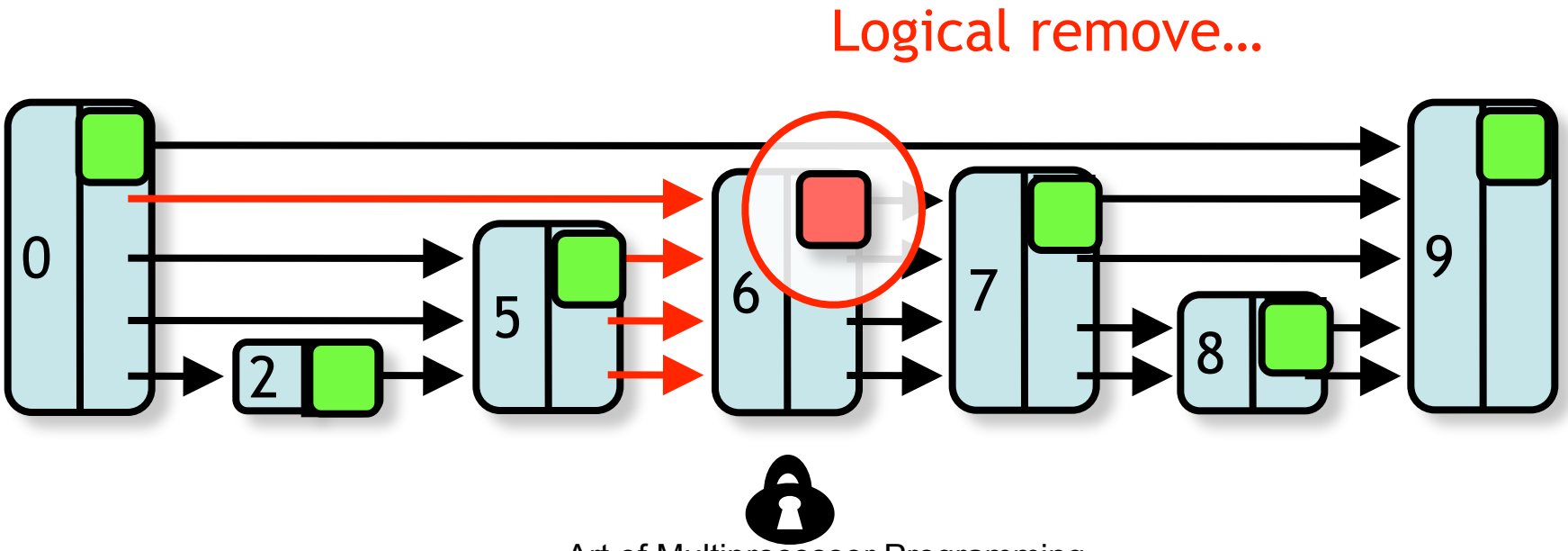
contains(8)

Prove: an unmarked node (like 8) remains reachable



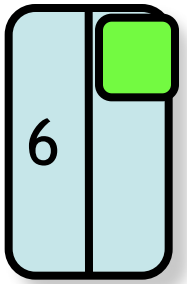
remove(6): Linearization

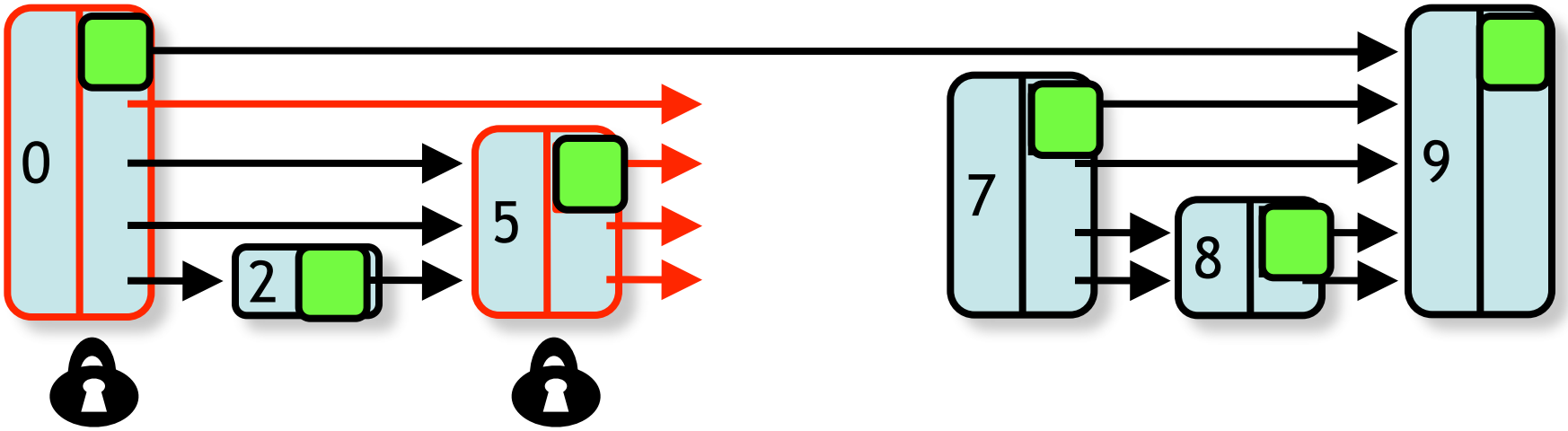
- Successful remove happens when bit is set



Add: Linearization

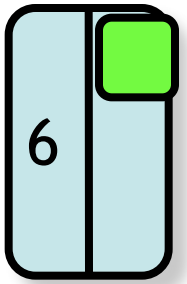
- Successful add() at point when **fully** linked
- Add fullyLinked bit to indicate this
- Bit tested by contains()

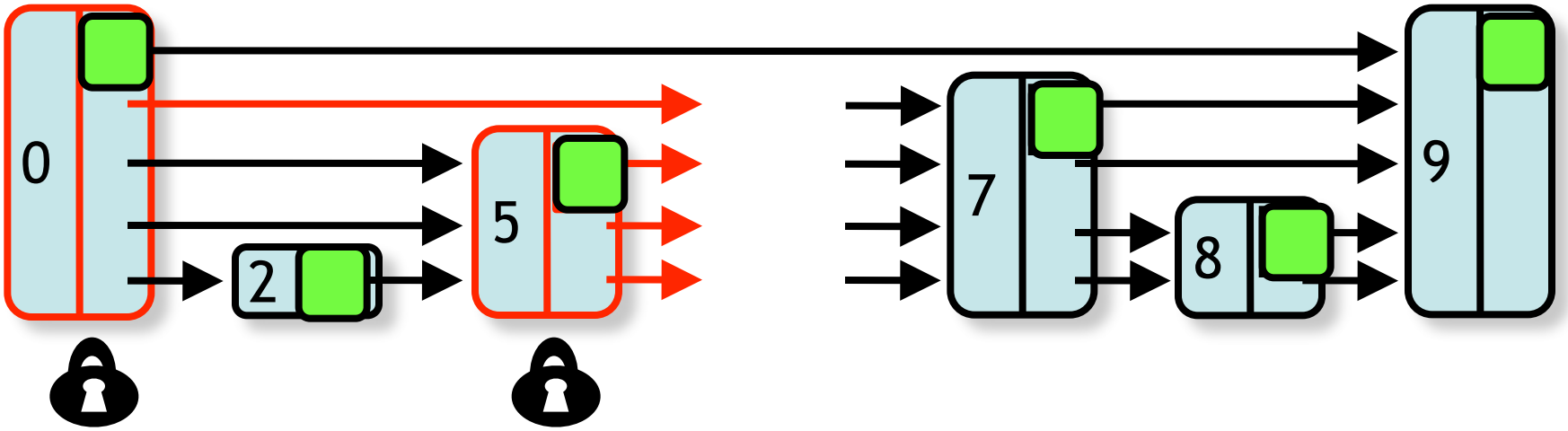




Add: Linearization

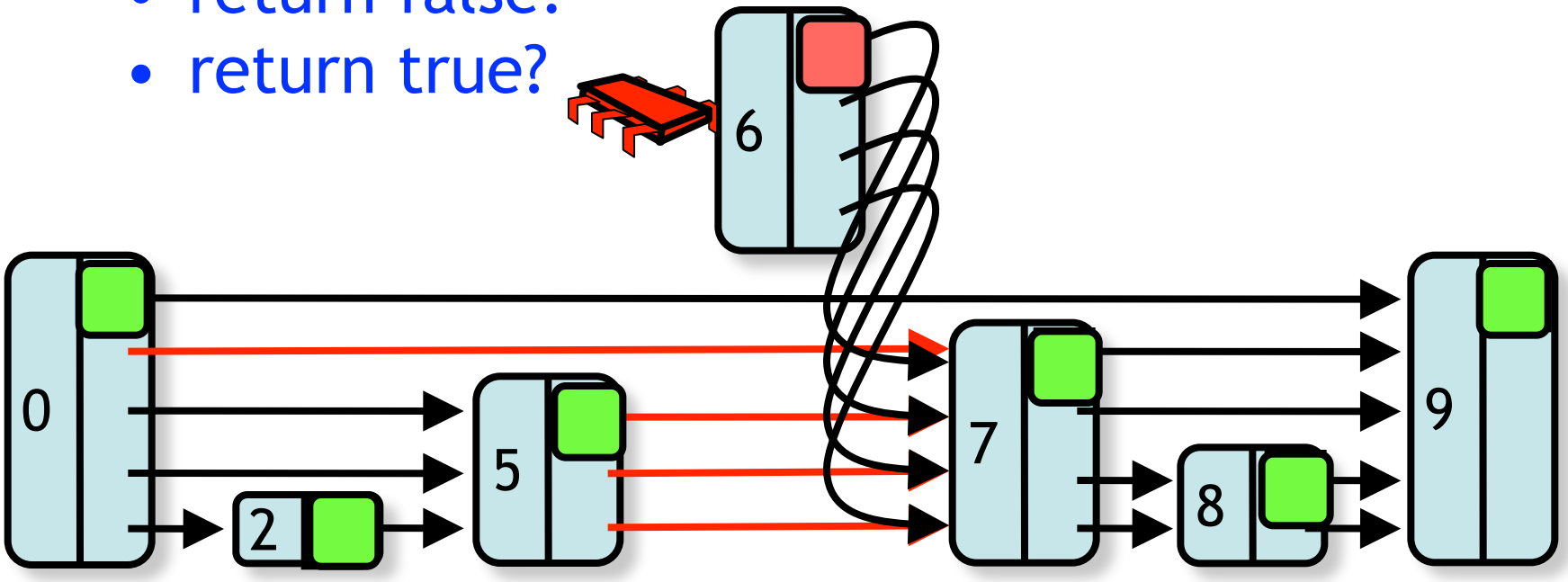
- Successful add() at point when **fully** linked
- Add fullyLinked bit to indicate this
- Bit tested by contains()





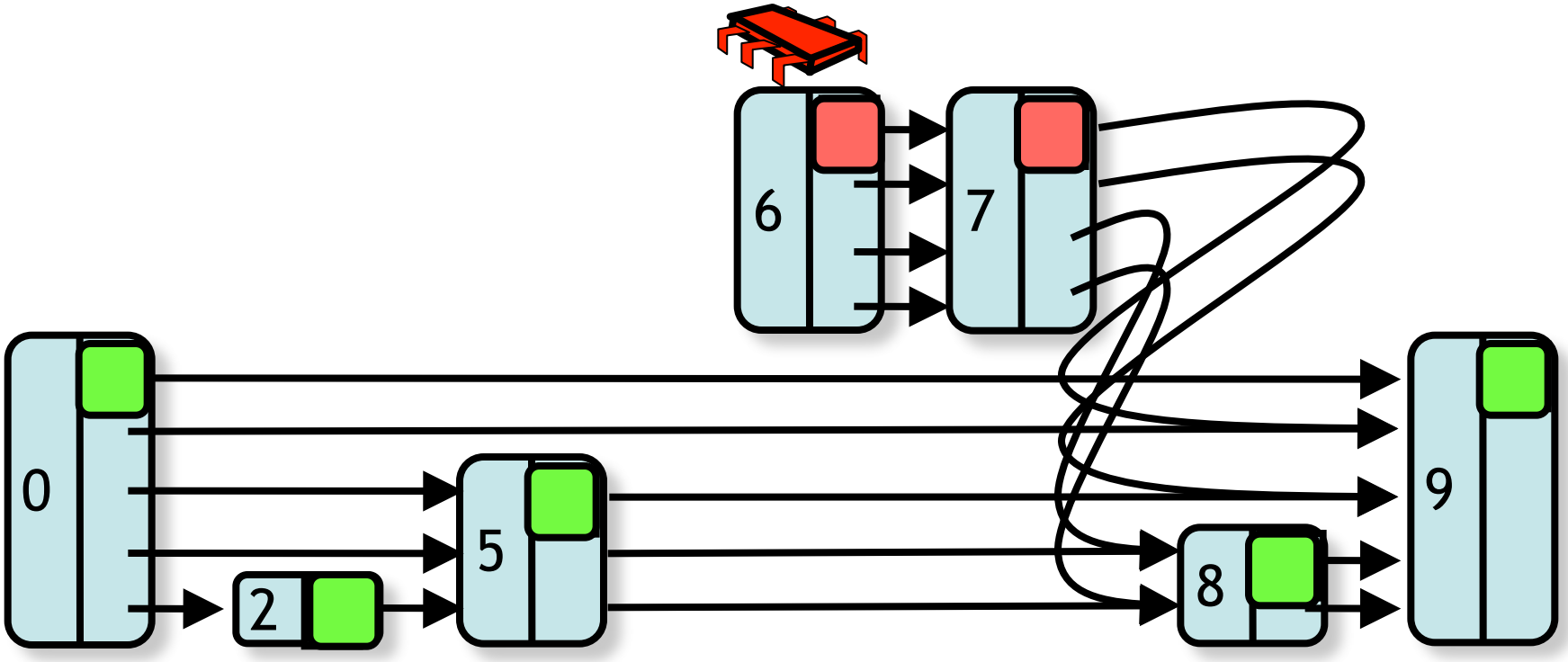
contains(7): Linearization

- When not fully-linked unmarked node found
  - pause while fullyLinked bit unset?
  - return false?
  - return true?



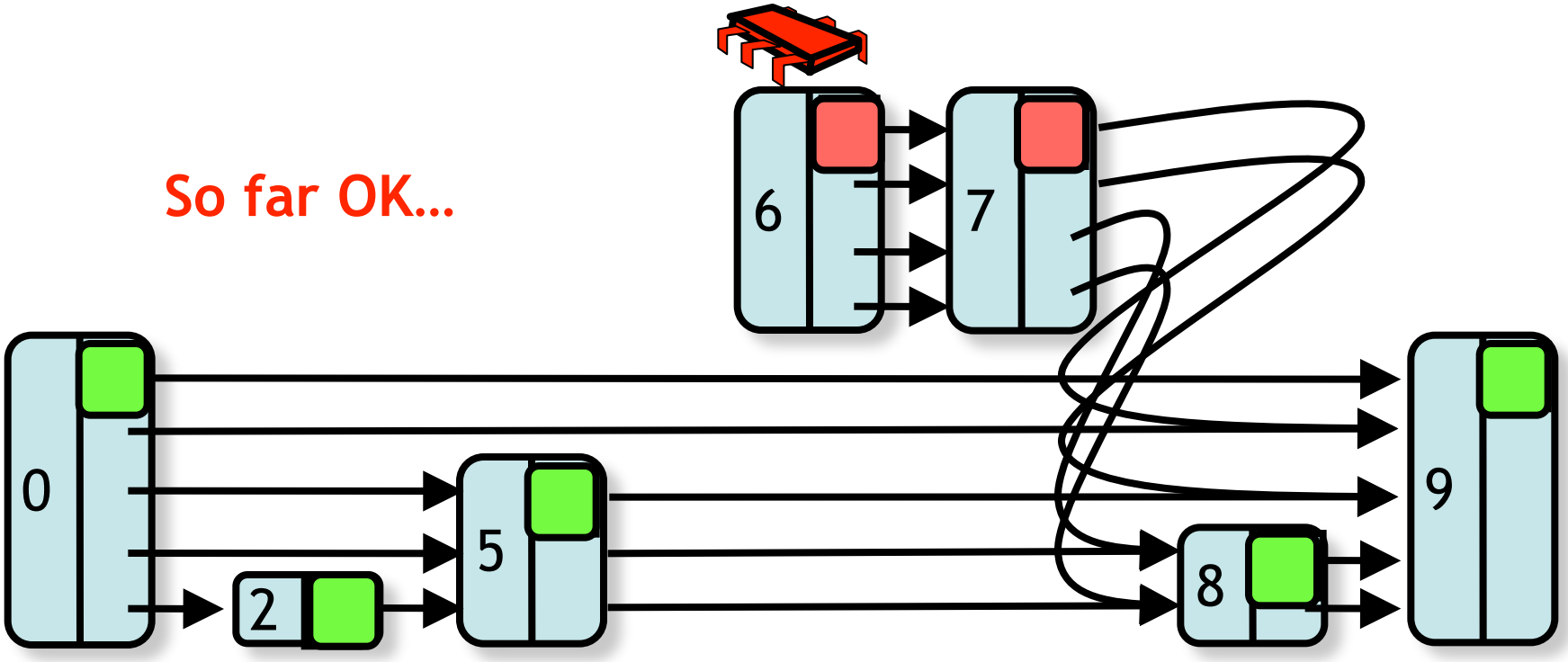
contains(7): Linearization

- When do we linearize unsuccessful Search?



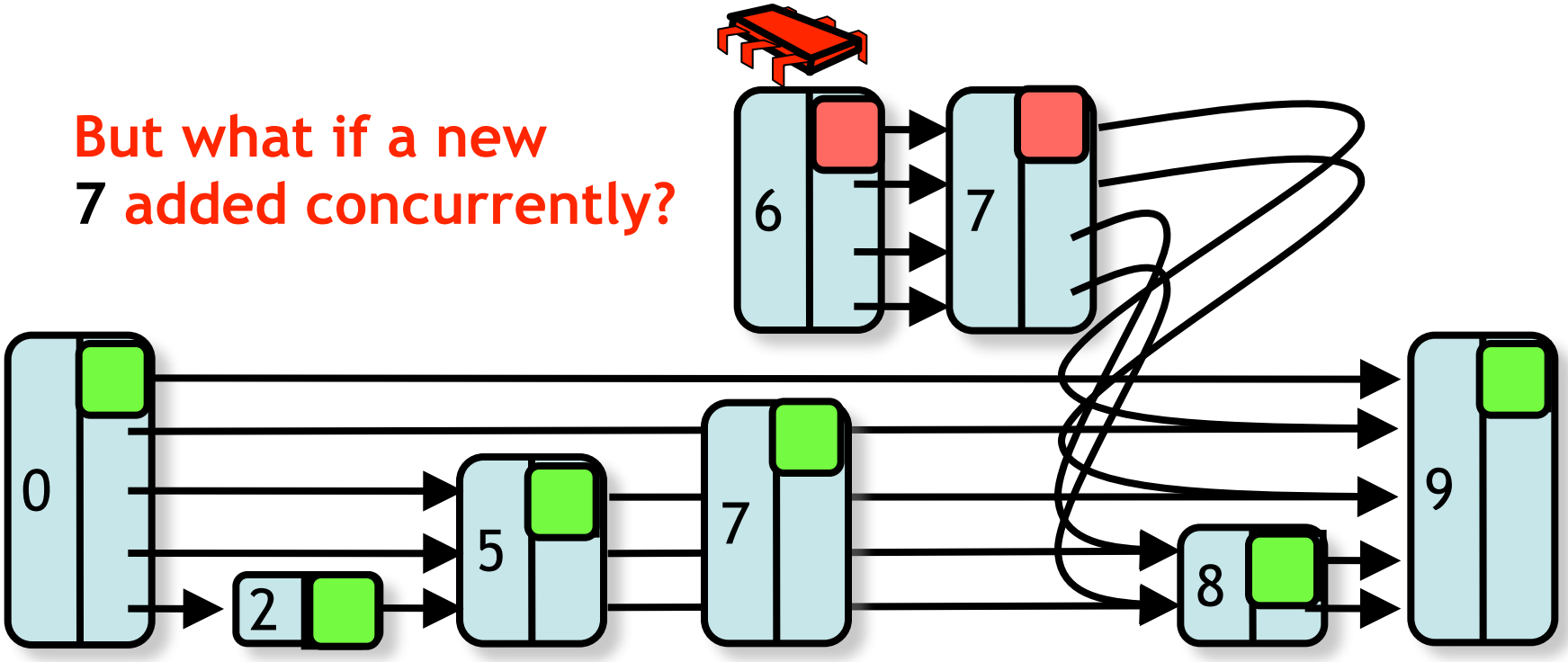
contains(7): Linearization

- When do we linearize unsuccessful Search?



contains(7): Linearization

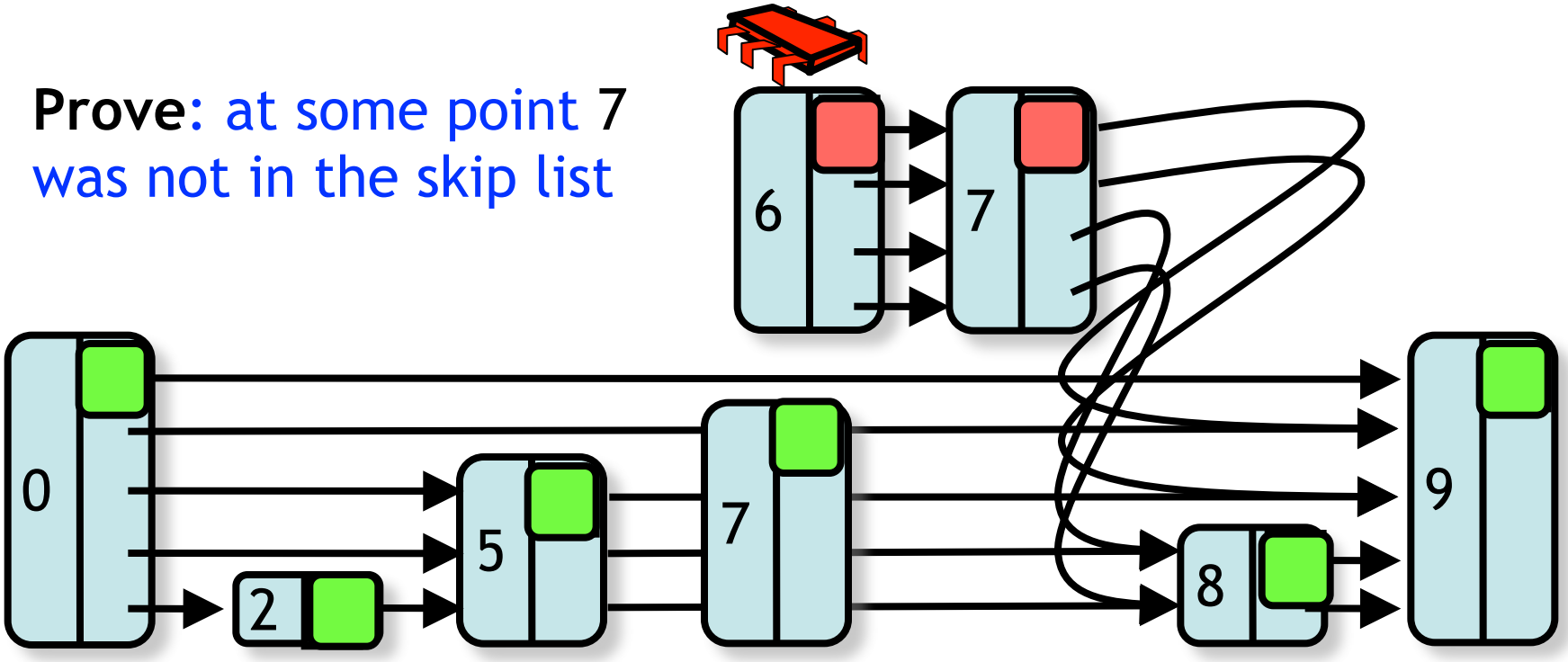
- When do we linearize unsuccessful Search?



contains(7): Linearization

- When do we linearize unsuccessful Search?

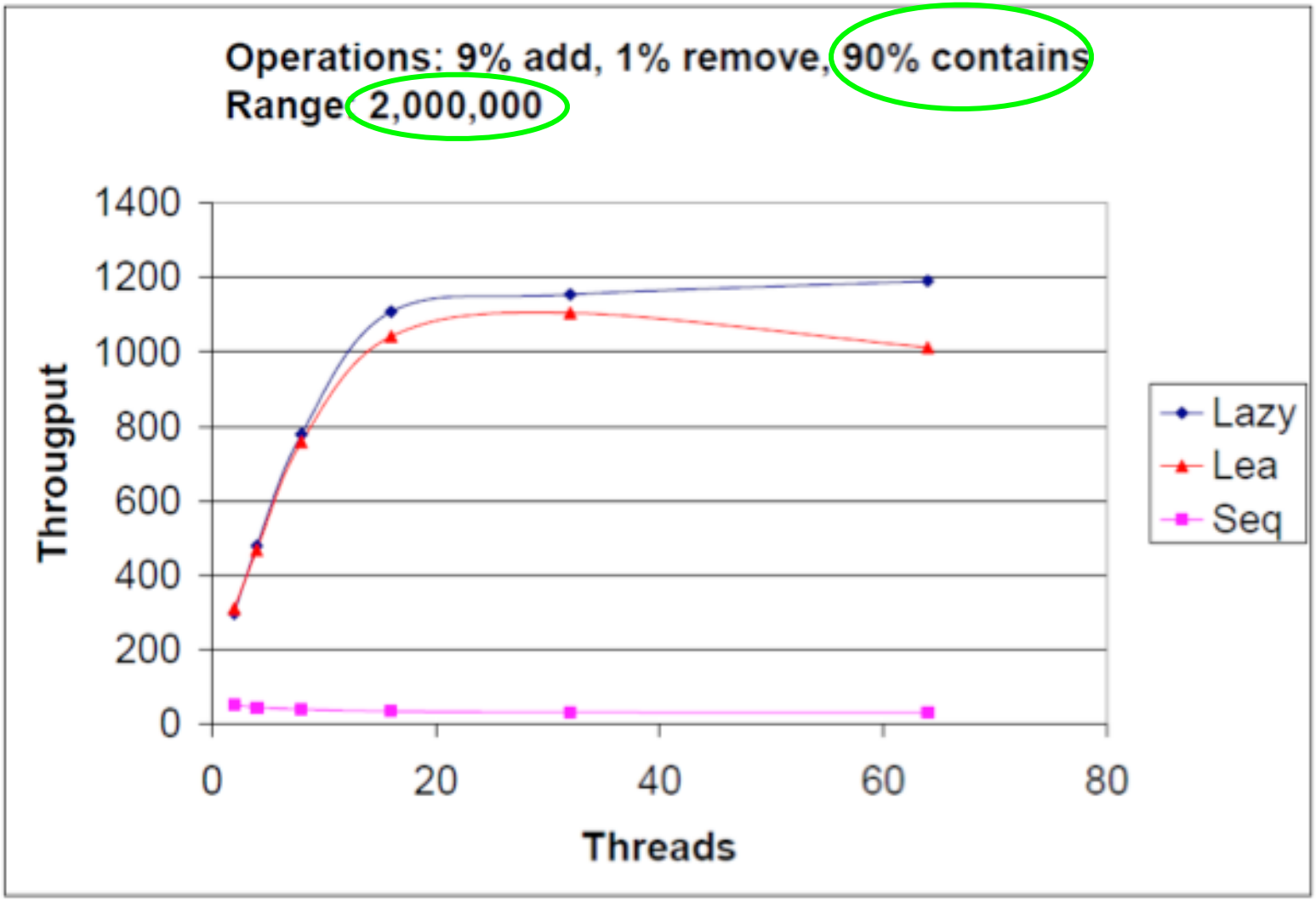
Prove: at some point 7 was not in the skip list



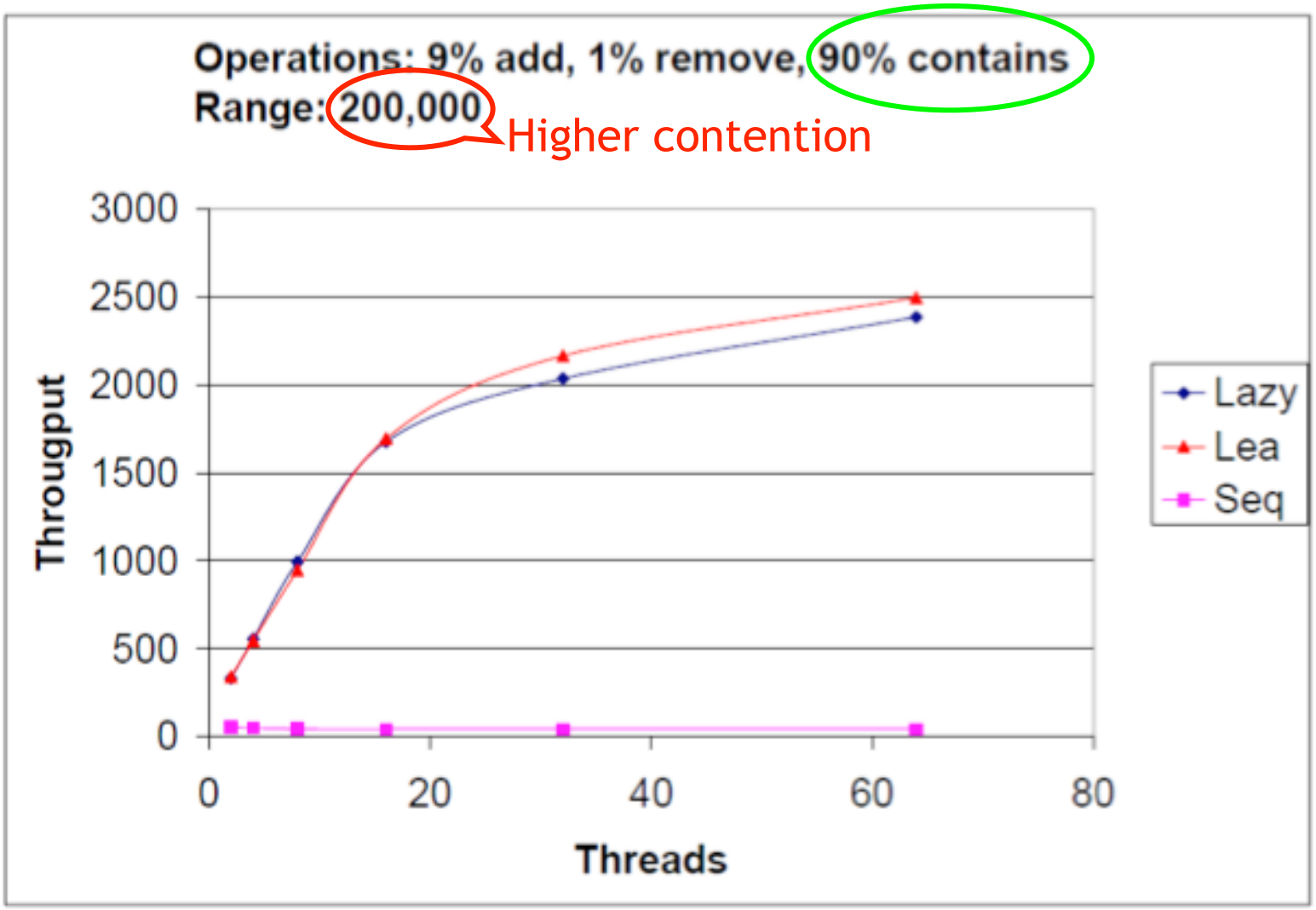
A Simple Experiment

- Each thread runs 1 million iterations, each either:
  - **add()**
  - **remove()**
  - **contains()**
- Item and method chosen in random from some distribution

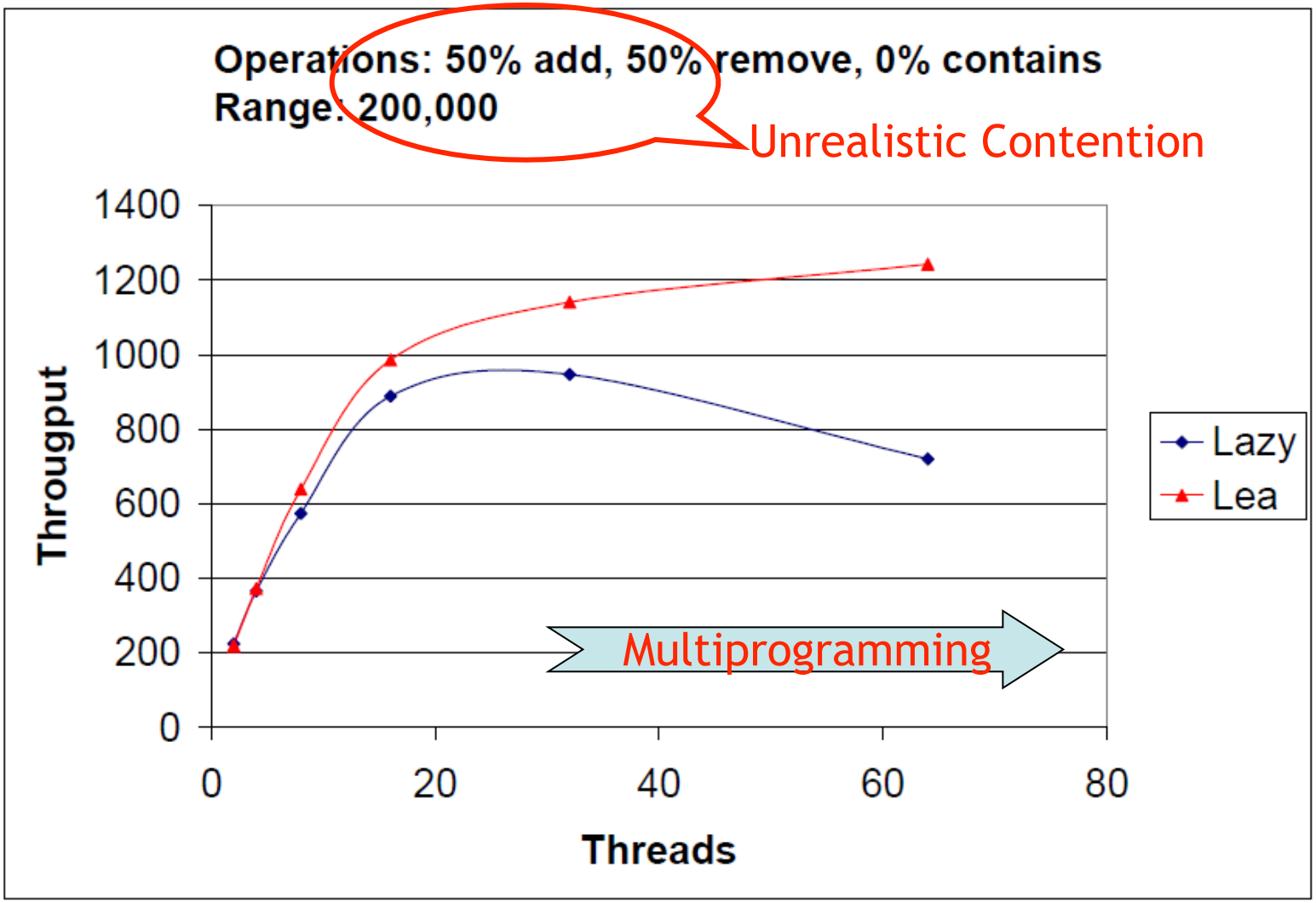
Lazy Skip List: Performance



Lazy Skip List: Performance



Lazy Skip List: Performance



Summary

- Lazy Skip List
  - Optimistic fine-grained Locking
- Performs as well as the lock-free solution in "common" cases
- Simple