The Universality of Consensus

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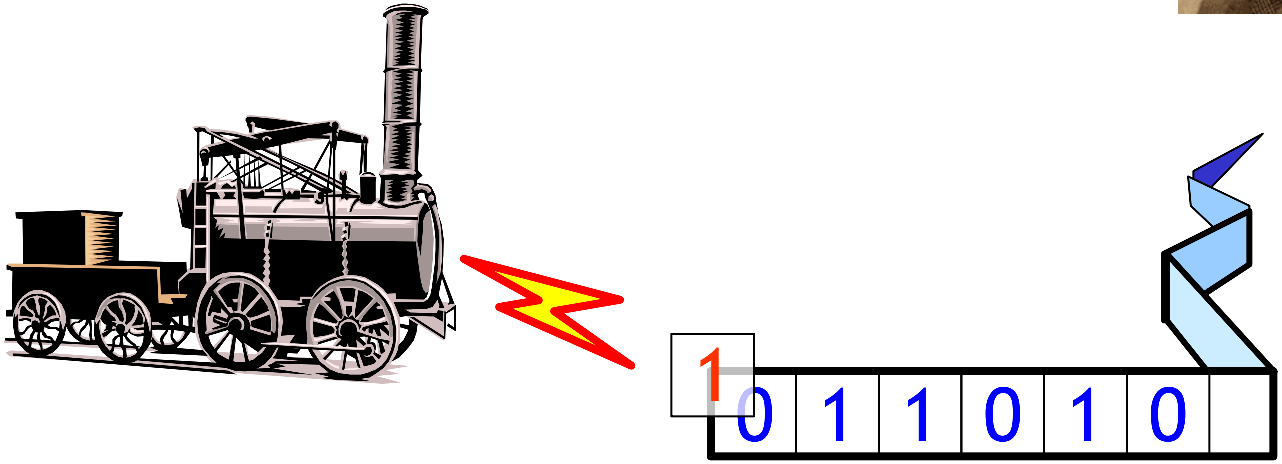
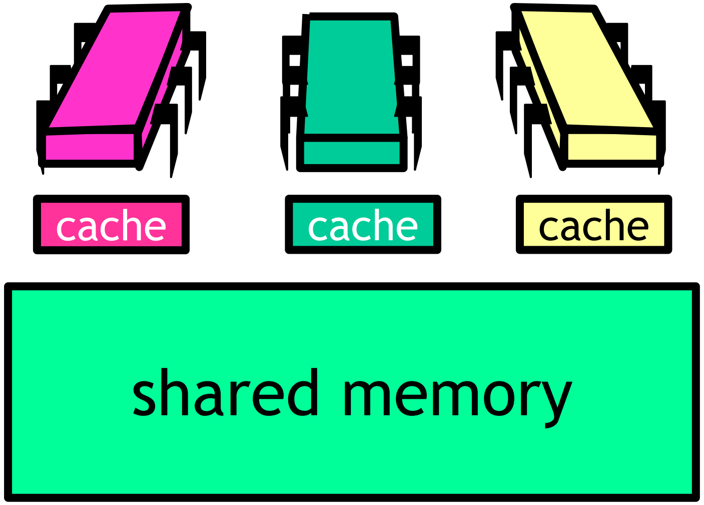
Based on slides by Maurice Herlihy and Nir Shavit

Turing Computability



- A mathematical model of computation
- Computable = Computable on a T-Machine

Shared-Memory Computability



- Model of asynchronous concurrent computation
- Computable = Wait-free/Lock-free computable on a multiprocessor

The Con

Can we implement them from any other object that has consensus number ∞?

Like compareAndSet()…

1 Read/W
2 g FIFO Queue, LIFO Stack
. . .
∞ c Multiple Assignment

Theorem: Universality

- Consensus is universal
- From n-thread consensus build a
  - Wait-free
  - Linearizable
  - n-threaded implementation
  - Of any sequentially specified object

Proof Outline

- A universal construction
  - From n-consensus objects
  - And atomic registers
- Any wait-free linearizable object
  - Not a practical construction
  - But we know where to start looking …

Like a Turing Machine

- This construction
  - Illustrates what needs to be done
  - Optimization fodder

- Correctness, not efficiency

- (I will also show you a more practical proposal)

A Generic Sequential Object

```
public interface SeqObject {
  public abstract
    Response apply(Invoc invoc);
}
```

A Generic Sequential Object

```
public interface SeqObject {
  public abstract
    Response apply(Invoc invoc);
}
```

Push:5, Pop:void

Invocation

```
public class Invoc {
 public String method;
 public Object[] args;
}
```

Invocation

```
public class Invoc {
 public String method;
 public Object[] args;
}
```

Method name

Invocation

```
public class Invoc {
 public String method;
 public Object[] args;
}
```

Arguments

A Generic Sequential Object

```
public interface SeqObject {
  public abstract Response apply(Invocation invoc);
}
```

OK, 4

Response

```
public class Response {
public Object value;
}
```

Return value

A Universal Concurrent Object

```
public interface SeqObject {
  public abstract
    Response apply(Invoc invoc);
}
```

A concurrent object that is linearizable to the generic sequential object

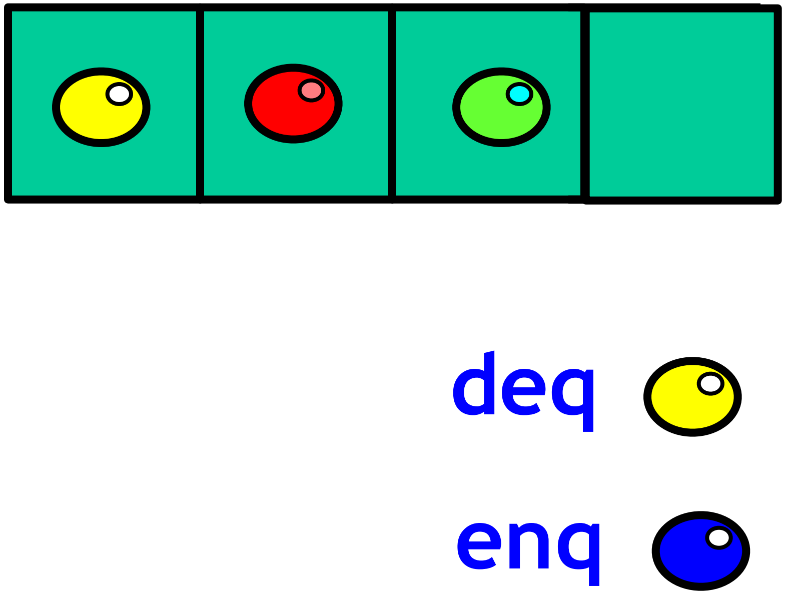
Start with Lock-Free Universal Construction

- First Lock-free: infinitely often some method call finishes.
- Then Wait-Free: each method call takes a finite number of steps to finish

Universal Construction: Naïve Idea

- Consensus object stores reference to cell with current state
- Each thread creates new cell
  - computes outcome,
  - and tries to switch pointer to its outcome
- Unfortunately not…
  - consensus objects can be used once only
  - might overwrite previous changes

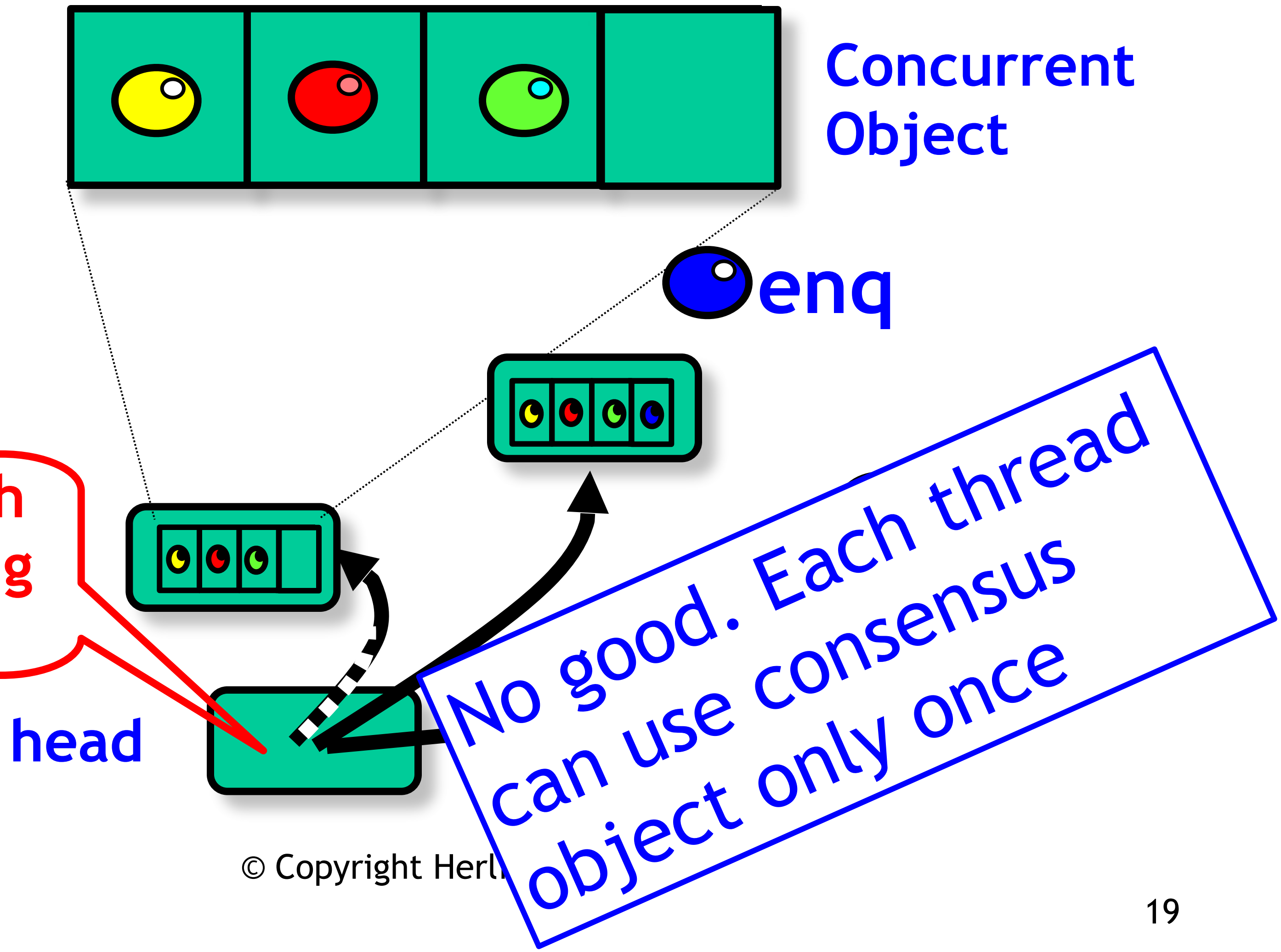
Naïve Idea



deq

enq

Naïve Idea



Decide which to apply using consensus

No good. Each thread can use consensus object only once

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Remarks

- Actually, not that bad if you use CAS
- To execute a method, a thread:
  - copies current state s
  - applies method - resulting in state s’
  - atomically replaces s by s’ using CAS
  - repeat if CAS fails

Why only once? Why is consensus object not readable?

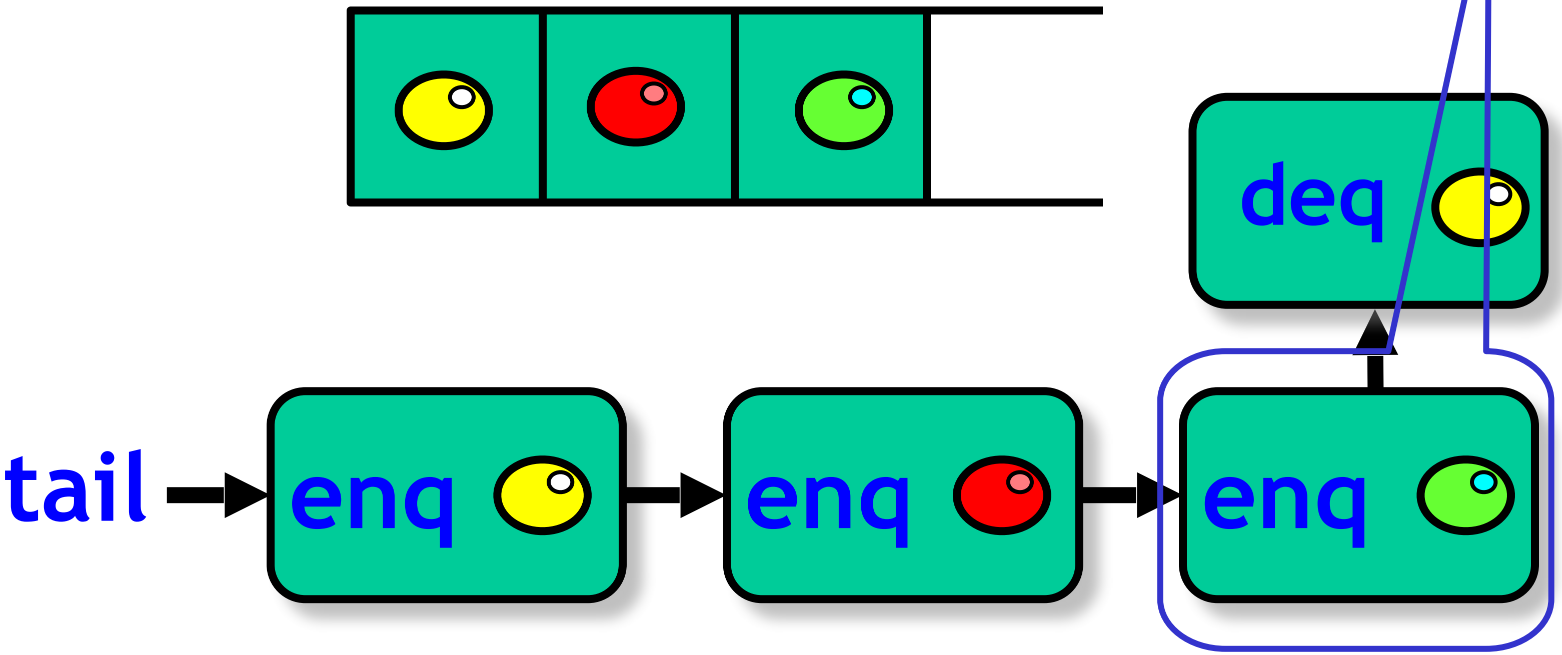
Queue based consensus

```
public decide(object value) {
  propose(value);
  Ball ball = this.queue.deq();
  if (ball == Ball.RED)
   return proposed[i];
  else
   return proposed[1-i];
}
```

Solved one time **2-consensus. Not clear how to allow reuse of object or reading its state…**

Improve
Re

Each node contains a fresh consensus object used to decide on next operation



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Universal Construction

- Object represented as
  - Initial Object State
  - A Log: a linked list of the method calls
- New method call
  - Find end of list
  - Atomically append call
  - Compute response by traversing the log up to the call

Basic Idea

- Use one-time consensus object to decide next pointer
- All threads update actual next pointer based on decision
  - OK because they all write the same value
- Challenges
  - Lock-free means we need to worry what happens if a thread stops in the middle

Basic Data Structures

```
public class Node implements java.lang.Comparable
{
 public Invoc invoc;
 public Consensus<Node> decideNext;
 public Node next;
 public int seq;
 public Node(Invoc inv) {
    invoc = inv;
    decideNext = new Consensus<Node>()
    seq = 0;
  }
```

Basic Data Structures

```
public class Node implements java.lang.Comparable
{
 public Invoc invoc;
 public Consensus<Node> decideNext;
 public Node next;
 public int seq;
 public Noc
   invoc = i
   decideNext = new Consensus<Node>()
   seq = 0;
 }
```

Standard interface for class whose objects are totally ordered

Basic Data Structures

```
public class Node implements java.lang.Comparable
{
 public Invoc invoc;
 public Consensus<Node> decideNext;
 public Node next;
 public int seq;
 public Node(Invoc invoc) {
   invoc = inv;
   decideNext = new Consensus<Node>()
   seq = 0;
 }
```

the invocation

Basic Data Structures

```
public class Node implements java.lang.Comparable
{
 public Invoc invoc;
 public Consensus<Node> decideNext;
 public Node next;
 public int seq;
 public Node(Invoc inv) {
   invoc = inv;
   decideNext = new Consensus<Node>()
   seq = 0;
 }
```

Decide on next node (next method applied to object)

Basic Data Structures

```
public class Node implements java.lang.Comparable
{
 public Invoc invoc;
 public Consensus<Node> decideNext;
 public Node next;
 public int seq;
 public Node(Invoc inv) {
   invoc = inv;

 }
```

Traversable pointer to next node (needed because you cannot repeatedly read a consensus object)

Basic Data Structures

```
public class Node implements java.lang.Comparable
{
 public Invoc invoc;
 public Consensus<Node> decideNext;
 public Node next;
 public int seq;
 public Node(Invoc inv) {
   invoc = invoc;
   decideNext = new Consensus<Node>()
   seq = 0;
  }
```

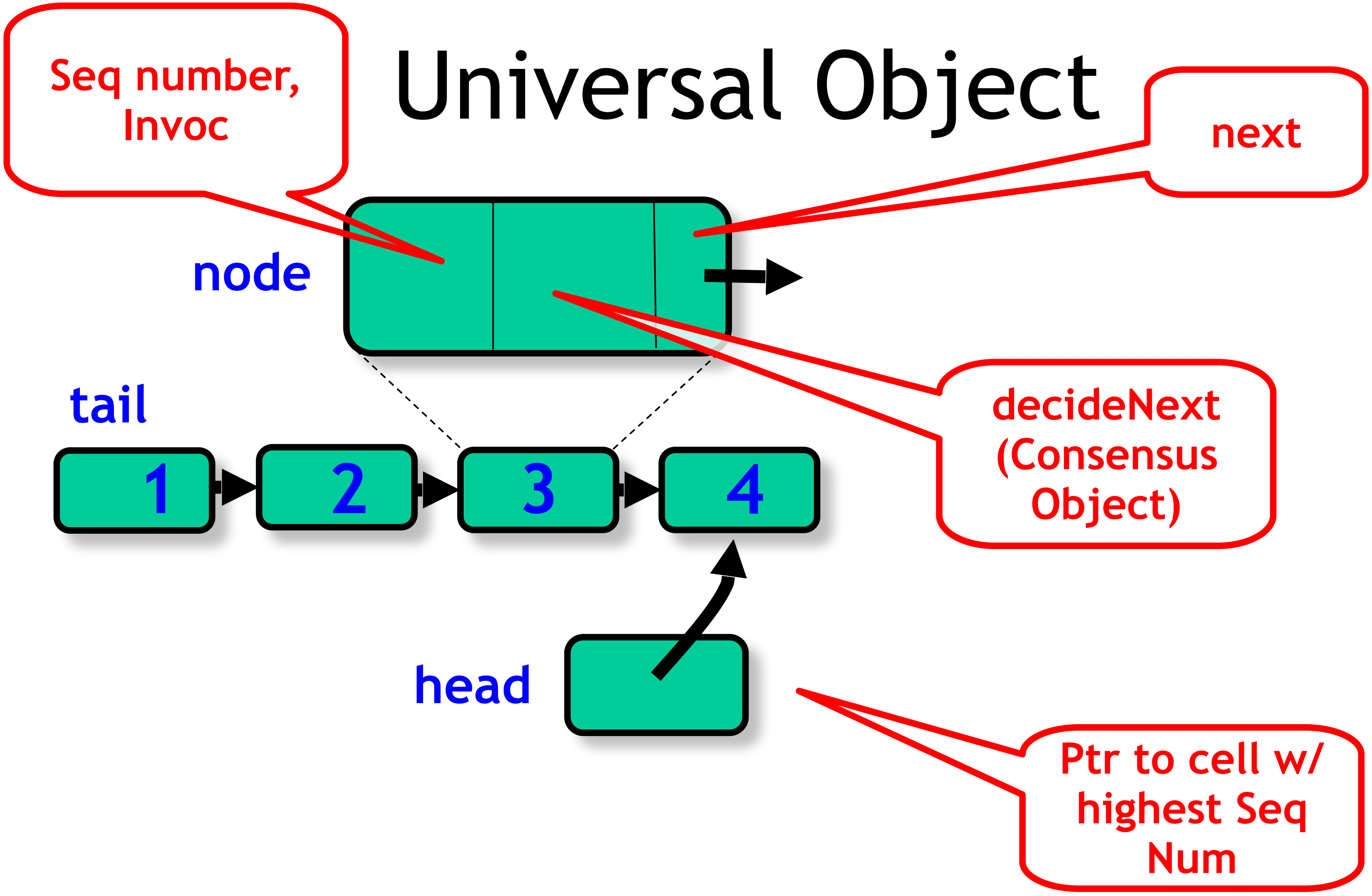
Seq number

Basic Data Structures

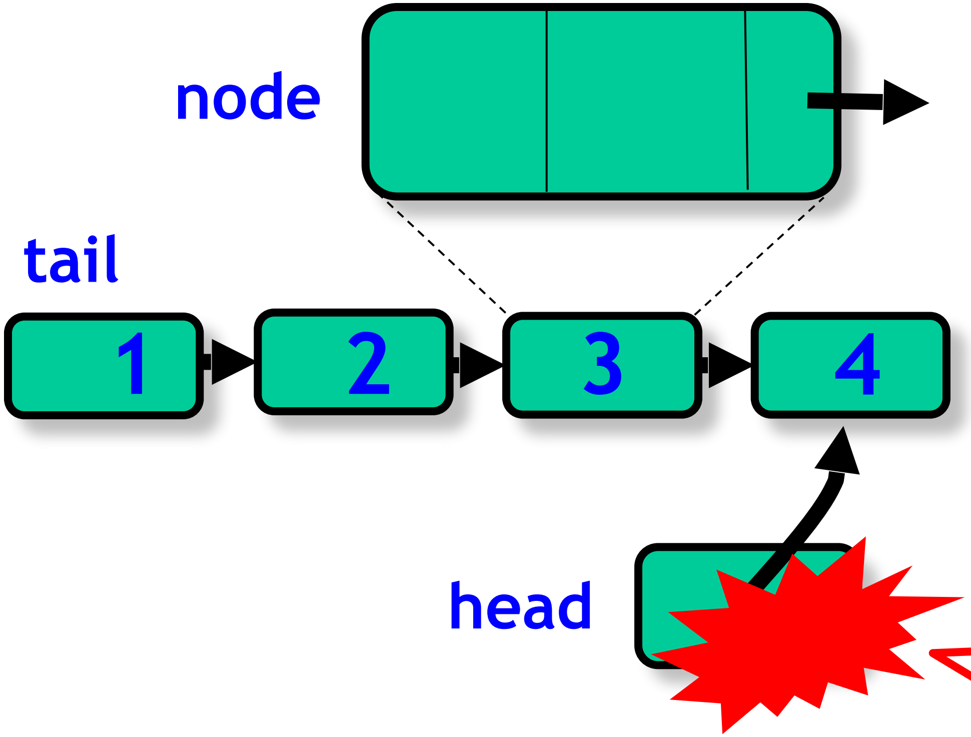
```
public class Node implements java.lang.Comparable
{
 p
 p
 public Node next;
 public int seq;
 public Node(Invoc inv) {
   invoc = inv;
   decideNext = new Consensus<Node>();
   seq = 0;
  }
```

Create a new node for a given method invocation

Universal Object



Universal Object



All threads repeatedly modify head… back to where we started?

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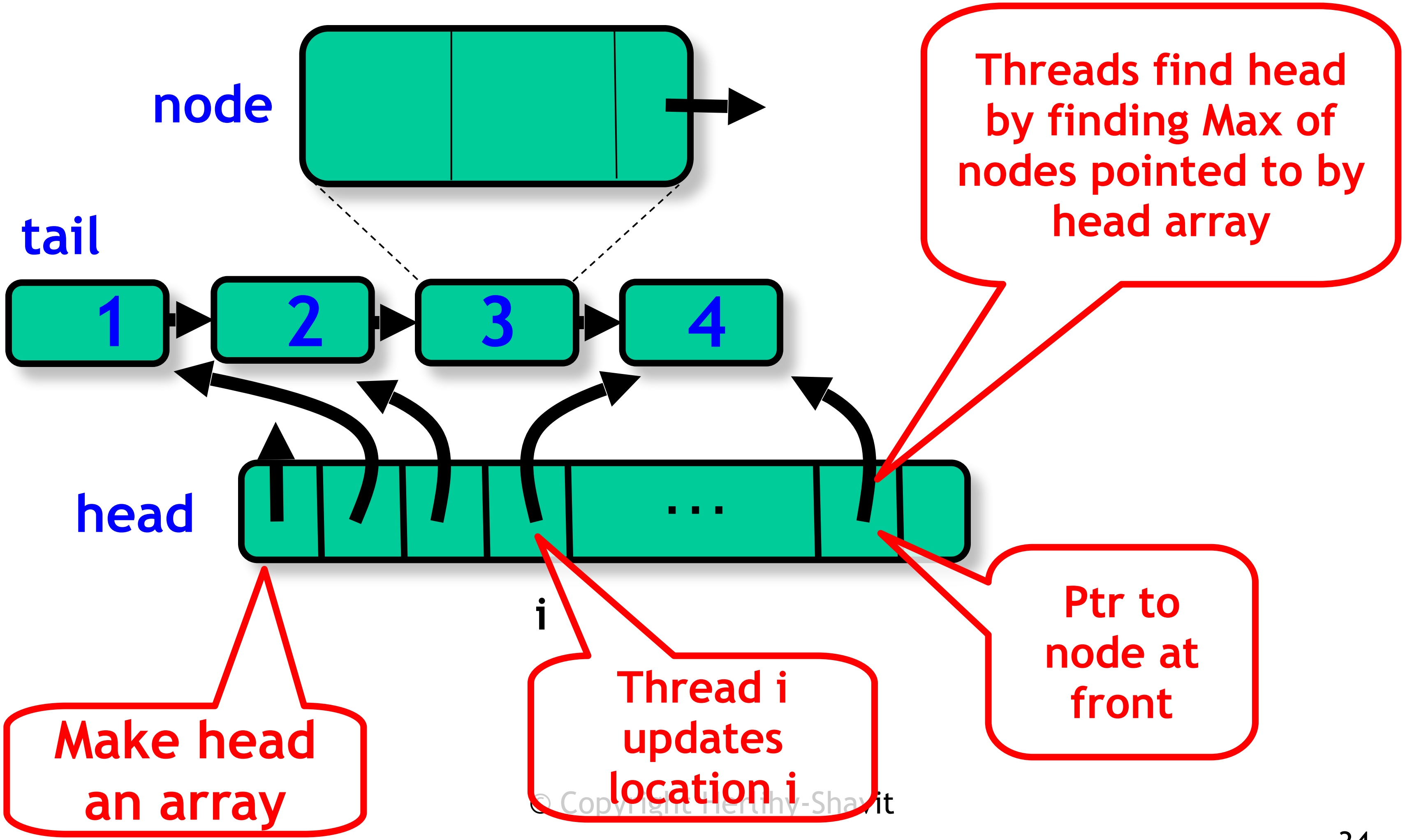
The Solution



Threads find head by finding Max of nodes pointed to by head array

Ptr to node at front

Thread i updates location i

Make head an array

Universal Object

```
public class Universal {
  private Node[] head;
  private Node tail = new Node();
  tail.seq = 1;
  for (int j=0; j < n; j++){
          head[j] = tail
     }
```

Universal Object

```
public class Universal {
  private Node[] head;
  private Node tail = new Node();
  tail.seq = 1;
  for (int j=0; j < n; j++){
         head[j] = tail
    }
```

Head Pointers Array

Universal Object

```
public class Universal {
  private Node[] head;
  private Node tail = new Node();
  tail.seq = 1;
  for (int j=0; j < n; j++){
        head[j] = tail
    }
```

Tail is a sentinel node with sequence number 1

Universal Object

```
public class Universal {
  private Node[] head;
  private Node tail = new Node();
  tail.seq = 1;
  for (int j=0; j < n; j++){
         head[j] = tail
     }
```

Initially head points to tail

Find Max Head Value

```
public static Node max(Node[] array) {
    Node max = array[0];
    for (int i = 1; i < array.length; i++)
      if (max.seq < array[i].seq)
        max = array[i];
      return max;
  }
```

Find Max Head Value

```
public static Node max(Node[] array) {
  Node max = array[0];
  for (int i = 1; i < array.length; i++)
   if (max.seq < array[i].seq)
    max = array[i];
   return max;
 }
```

Traverse the array

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Find Max Head Value

```
public static Node max(Node[] array) {
    Node max = array[0];
    for (int i = 1; i < array.length; i++)
      if (max.seq < array[i].seq)
        max = array[i];
      return max;
  }
```

Compare the seq nums of nodes pointed to by the array

Find Max Head Value

```
public static Node max(Node[] array) {
    Node max = array[0];
    for (int i = 1; i < array.length; i++)
      if (max.seq < array[i].seq)
        max = array[i];
      return max;
  }
```

return the node with max. seq number

Universal Application Part I

```
public Response apply(Invoc invoc) {
  int i = ThreadID.get();
  Node prefer = new node(invoc);
  while (prefer.seq == 0) {
    Node before = Node.max(head);
    Node after =
     before.decideNext.decide(prefer);
    before.next = after;
    after.seq = before.seq + 1;
    head[i] = after;
    }
  …
```

Universal Application Part I

```
public Response apply(Invoc invoc) {
  int i = ThreadID.get();
  Node prefer = new node(invoc);
  while (prefer.seq == 0) {
    Node before = Node.max(head);
    Node after =
     before.decideNext.decide(prefer);
    before.next = after;
    after.seq = before.seq + 1;
    head[i] = after;
  }
  …
```

Apply will have invocation as input and return the appropriate response

Universal Application Part I

```
public Response apply(Invoc invoc) {
  int i = ThreadID.get();
  Node prefer = new node(invoc);
  while (prefer.seq == 0) {
    Node before = Node.max(head);
    Node after =
     before.decideNext.decide(prefer);
    before.next = after;
    after.seq = before.seq + 1;
    head[i] = after;
  }
  …
```

My id

Universal Application Part I

```
public Response apply(Invoc invoc) {
 int i = ThreadID.get();
 Node prefer = new node(invoc);
 while (prefer.seq == 0) {
   Node before = Node.max(head);
   Node after =
    before.decideNext.decide(prefer);
   before.next = after;
   after.seq = before.seq + 1;
   head[i] = after;
   }
 …
```

My method call

Universal Application Part I

```
public Response apply(Invoc invoc) {
  int i = ThreadID.get();
  Node prefer = new node(invoc);
  while (prefer.seq == 0) {
    Node before = Node.max(head);
    Node after =
     before.decideNext.decide(prefer);
    before.next = after;
    after.seq = before.seq + 1;
    head[i] = after;
  }
…
```

As long as I have not been threaded into list

Universal Application Part I

```
public Response apply(Invoc invoc) {
  int i = ThreadID.get();
  Node prefer = new node(invoc);
  while (prefer.seq == 0) {
    Node before = Node.max(head);
    Node after =
     before.decideNext.decide(prefer);
    before.next = after;
    after.seq = before.seq + 1;
    head[i] = after;
    }
  …
```

Node at head of list that we will try to append to

Universal Application Part I

```
public Response apply(Invoc invoc) {
  int i = ThreadID.get();
  Node prefer = new node(invoc);
  while (prefer.seq == 0) {
    Node before = Node.max(head);
    Node after =
     before.decideNext.decide(prefer);
    before.next = after;
    after.seq = before.seq + 1;
    head[i] = after;
  }
}
```

Decide winning node; could have already been decided

Universal Application

```
public Response apply(Invoc invoc) {
  int i = ThreadID.get();
  Node prefer = new node(invoc);
  while (prefer.seq == 0) {
    Node before = Node.max(head);
    Node after =
     before.decideNext.decide(prefer);
    before.next = after;
    after.seq = before.seq + 1;
    head[i] = after;
  }
}
```

Could have already been set by winner…in which case no effect

Set next pointer based on decision

Universal Application Part I

```
public Response apply(Invoc invoc) {
  int i = ThreadID.get();
  Node prefer = new node(invoc);
  while (prefer.seq == 0) {
    Node before = Node.max(head);
    Node after =
     before.decideNext.decide(prefer);
    before.next = after;
    after.seq = before.seq + 1;
    head[i] = after;
  }
…
```

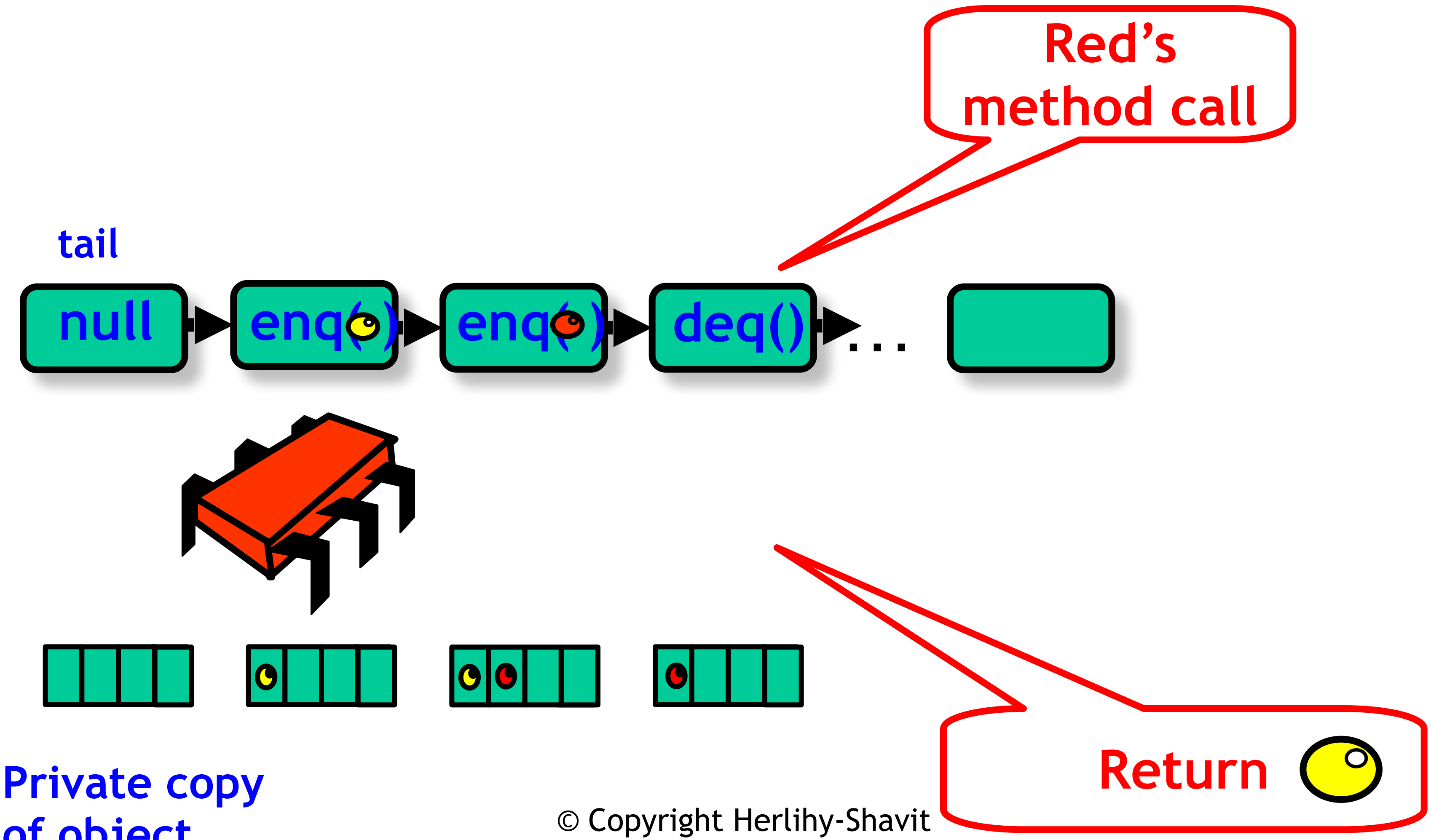
Set seq number indicating node was appended

Universal Application Part I

```
public Response apply(Invoc invoc) {
  int i = ThreadID.get();
  Node prefer = new node(invoc);
  while (prefer.seq == 0) {
    Node before = Node.max(head);
    Node after =
     before.decideNext.decide(prefer);
    before.next = after;
    after.seq = before.seq + 1;
    head[i] = after;
    }
  …
```

add to head array so new head will be found

Part II – Compute Response



Private copy of object

Universal Application Part II

```
...
//compute my response
SeqObject MyObject = new SeqObject();
current = tail.next;
while (current != prefer) {
    MyObject.apply(current.invoc);
    current = current.next;
}
return MyObject.apply(current.invoc);
```

Universal Application Part II

```
...
//compute my response
SeqObject MyObject = new SeqObject();
current = tail.next;
while (current != prefer){
     MyObject.apply(current.invoc);
     current = current.next;
  }
```

Compute the result by sequentially applying the method calls in the list to a private copy of the object starting from the initial state

Universal Application Part II

```
...
//compute my response
SeqObject MyObject = new SeqObject();
current = tail.next;
while (current != prefer){
     MyObject.apply(current.invoc);
     current = current.next;
}
return MyObject.apply(current.invoc);
```

Start with initialized copy of the sequential object

Universal Application Part II

```
...
//compute my response
SeqObject MyObject = new SeqObject();
current = tail.next;
while (current != prefer){
     MyObject.apply(current.invoc);
     current = current.next;
}
return MyObject.apply(current.invoc);
```

First method call is appended after the tail

Universal Application Part II

```
...
//compute my response
SeqObject MyObject = new SeqObject();
current = tail.next;
while (current != prefer){
    MyObject.apply(current.invoc);
    current = current.next;
}
return MyObject.apply(current.invoc);
```

While not reached my own method call

Universal Application Part II

```
...
//compute my response
SeqObject MyObject = new SeqObject();
current = tail.next;
while (current != prefer){
   MyObject.apply(current.invoc);
   current = current.next;
}
return MyObject.apply(current.invoc);
```

Apply the current nodes method to object

Universal Application Part II

```
...
//compute my response
SeqObject MyObject = new SeqObject();
current = tail.next;
while (current != prefer){
    MyObject.apply(current.invoc);
    current = current.next;
}
return MyObject.apply(current.invoc);
```

Return the result after applying my own method call

Correctness

- List defines linearized sequential history
- Thread returns its response based on list order

Lock-freedom

- Lock-free because
- New winner node is added into the head array within a finite number of steps
- A thread moves forward in list
- Can repeatedly fail to win consensus on “real” head only if another succeeds

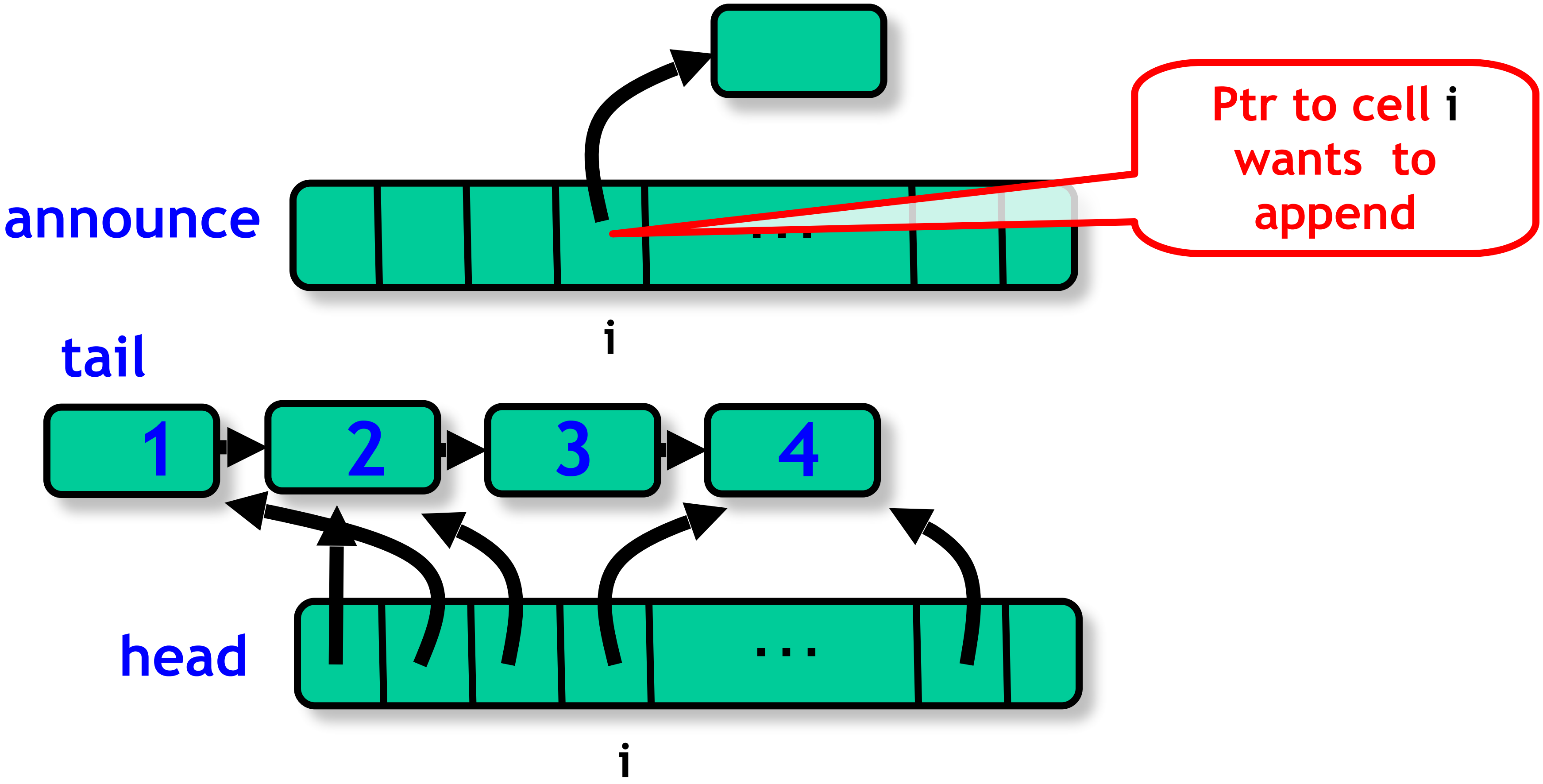
Wait-free Construction

- Lock-free construction + announce array
- Stores (pointer to) node in announce
  - If a thread doesn’t append its node
  - Another thread will see it in array and **help** append it

Helping

- “Announcing” my intention
  - Guarantees progress
  - Even if the scheduler hates me
  - My method call will complete
- Makes protocol wait-free
- Otherwise starvation possible
- Common in wait-free algorithms, but also used by lock-free implementations

Wait-free Construction



The Announce Array

```
public class Universal {
  private Node[] announce;
  private Node[] head;
  private Node tail = new node();
  tail.seq = 1;
  for (int j=0; j < n; j++){
    head[j] = tail; announce[j] = tail
  };
```

The Announce Array

```
public class Universal {
  private Node[] announce;
  private Node[] head;
  private Node tail = new node();
  tail.seq = 1;
  for (int j=0; j < n; j++){
    head[j] = tail; announce[j] = tail
  };
```

Announce array

The Announce Array

```
public class Universal {
  private Node[] announce;
  private Node[] head;
  private Node tail = new node();
  tail.seq = 1;
  for (int j=0; j < n; j++){
    head[j] = tail; announce[j] = tail
  };
```

All entries initially point to tail

A Cry For Help

```
public Response apply(Invoc invoc) {
  int i = ThreadID.get();
  announce[i] = new Node(invoc);
  head[i] = Node.max(head);
  while (announce[i].seq == 0) {
  …
  // while node not appended to list
  …
  }
```

A Cry For Help

```
public Response apply(Invoc invoc) {
  int i = ThreadID.get();
  announce[i] = new Node(invoc);
  head[i] = Node.max(head);
  while (announce[i].seq == 0) {
  …
  // while node not appended to list
  …
  }
```

Announce new method call (node), asking help from others

A Cry For Help

```
public Response apply(Invoc invoc) {
  int i = ThreadID.get();
  announce[i] = new Node(invoc);
  head[i] = Node.max(head);
  while (announce[i].seq == 0) {
  …
  // while node not appended to list
  …
  }
```

Look for end of list

A Cry For Help

```
public Response apply(Invoc invoc) {
  int i = ThreadID.get();
  announce[i] = new Node(invoc);
  head[i] = Node.max(head);
  while (announce[i].seq == 0) {
  …
  // while node not appended to list
  …
  }
```

Main loop, while node not appended (either by me or some thread helping me)

Main Loop

- Non-zero sequence number indicates success
- Thread keeps helping append nodes
- Until its own node is appended

Main Loop

```
while (announce[i].seq == 0) {
 Node before = head[i];
 Node help = announce[(before.seq + 1) % n];
 if (help.seq == 0)
     prefer = help;
    else
     prefer = announce[i];
…
```

Main Loop

```
while (announce[i].seq == 0) {
 Node before = head[i];
 Node help = announce[(before.seq + 1) % n];
 if (help.seq == 0)
     prefer = help;
    else
```

Keep trying until my cell gets a sequence number

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Main Loop

```
while (announce[i].seq == 0) {
 Node before = head[i];
 Node help = announce[(before.seq + 1) % n];
 if (help.seq == 0)
     prefer = help;
    else
     prefer = announce[i];
```

Possible end of list

Main Loop

```
while (announce[i].seq == 0) {
 Node before = head[i];
 Node help = announce[(before.seq + 1) % n];
 if (help.seq == 0)
     prefer = help;
    else
     prefer = announce[i];
```

Who do I help?

Altruism

- Choose a thread to “help”
- If that thread needs help
  - Try to append its node
  - Otherwise append your own
- Worst case
  - Everyone tries to help same pitiful loser
  - Someone succeeds

Help!

- When last node in list has sequence number k
- All threads check …
  - Whether thread k+1 mod n wants help
  - If so, try to append its node first

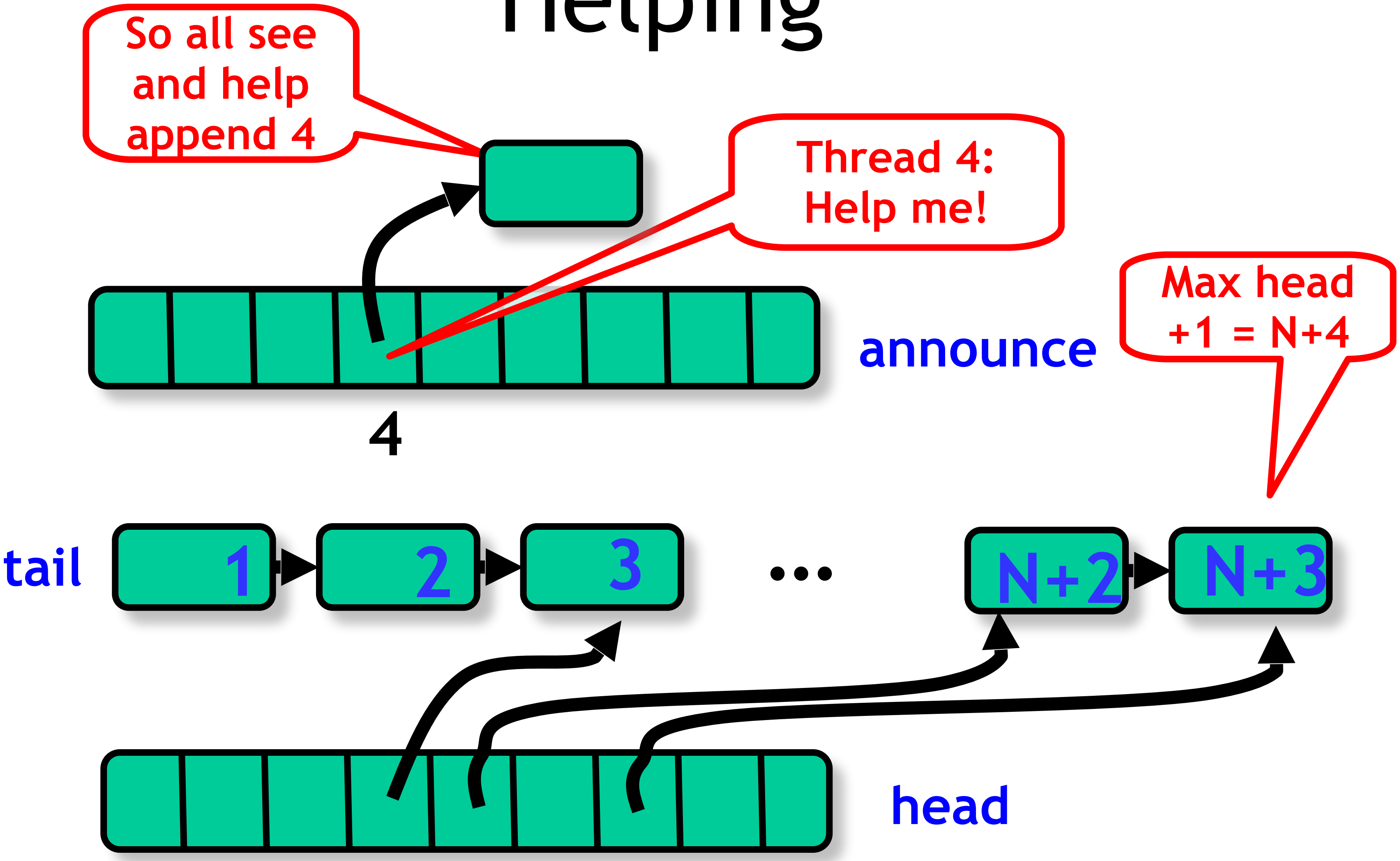
Help!

- First time after thread k+1 announces
  - No guarantees
- After at most n more nodes appended
  - Everyone sees that thread k+1 wants help
  - Everyone tries to append that node
  - Someone succeeds

Sliding Window Lemma

- After thread A announces its node
- No more than n other calls
  - Can start and finish
  - Without appending A's node

Helping



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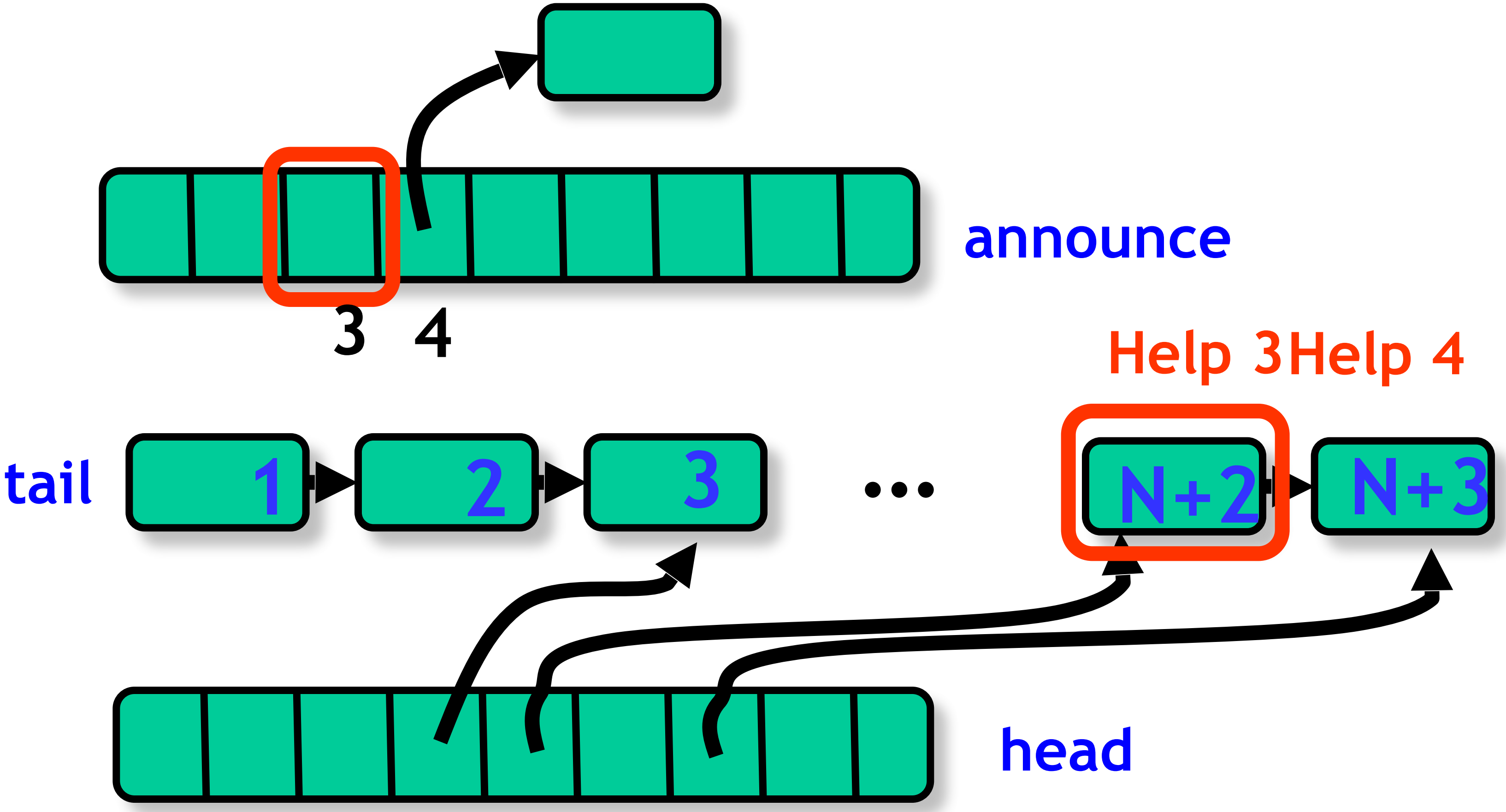
The Sliding Help Window



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Sliding Help Window

```
while (announce[i].seq == 0) {
 Node before = head[i];
 Node help = announce[(before.seq + 1) % n];
 if (help.seq == 0)
    prefer = help;
   else
    prefer = announce[i];
```

In each main loop iteration pick another thread to help

Sliding Help Window

Help if help required, but otherwise it’s all about me!

```
while (announce[i].seq == 0) {
 Node before = head[i];
 Node help = announce[(before.seq + 1 % n)];
 if (help.seq == 0)
     prefer = help;
    else
     prefer = announce[i];
…
```

Rest is Same as Lock-free

```
while (prefer.seq == 0) {
 …
 Node after =
  before.decideNext.decide(prefer);
 before.next = after;
 after.seq = before.seq + 1;
 head[i] = after;
}
```

Rest is Same as Lock-free

```
while (prefer.seq == 0) {
 …
 Node after =
  before.decideNext.decide(prefer);
 before.next = after;
 after.seq = before.seq + 1;
 head[i] = after;
}
```

Decide next node to be appended

Rest is Same as Lock-free

```
while (prefer.seq == 0) {
 …
 Node after =
  before.decideNext.decide(prefer);
 before.next = after;
 after.seq = before.seq + 1;
 head[i] = after;
}
```

Update next based on decision

Rest is Same as Lock-free

```
while (prefer.seq == 0) {
 …
 Node after =
  before.decideNext.decide(prefer);
 before.next = after;
 after.seq = before.seq + 1;
 head[i] = after;
}
```

Tell world that node is appended

Finishing the Job

- Once thread’s node is linked
- The rest is again the same as in lock-free algorithm
- Compute the result by sequentially applying the method calls in the list to a private copy of the object starting from the initial state

Then Same Part II

```
...
//compute my response
SeqObject MyObject = new SeqObject();
current = tail.next;
while (current != prefer){
    MyObject.apply(current.invoc);
    current = current.next;
 }
return MyObject.apply(current.invoc);
}
```

Universal Application Part II

```
...
//compute my response
SeqObject MyObject = new SeqObject();
current = tail.next;
while (current != prefer){
    MyObject.apply(current.invoc);
    current = current.next;
  }
return MyObject.apply(current.invoc);
}
```

Return the result after applying my own method call

GetAndSet is not Universal

```
public class RMWRegister {
 private int value;
 public boolean getAndSet(int update)
 {
  int prior = this.value;
  this.value = update;
  return prior;
 }
}
```

GetAndSet is not Universal

```
public class RMWRegister {
 private int value;
 public boolean getAndSet(int update)
 {
  int prior = this.value;
  this.value = update;
  return prior;
 }
}
```

Consensus number 2

GetAndSet is not Universal

```
public class RMWRegister {
 private int value;
 public boolean getAndSet(int update)
 {
  int prior = this.value;
  this.value = update;
  return prior;
 }
}
```

Not universal for ≥ 3 threads

CompareAndSet is Universal

```
public class RMWRegister {
 private int value;
 public boolean
   compareAndSet(int expected,
                int update) {
  int prior = this.value;
  if (this.value == expected) {
   this.value = update;
   return true;
  }
 return false;
 }}
```

CompareAndSet is Universal

```
public class RMWRegister {
 private int value;
 public boolean
   compareAndSet(int expected,
                  int update) {
  int prior = this.value;
  if (this.value == expected) {
   this.value = update;
   return true;
  }
 return false;
 }}
```

Consensus number ∞

CompareAndSet is Universal

```
public class RMWRegister {
 private int value;
 public boolean
  compareAndSet(int expected,
               int update) {
  int prior = this.value;
  if (this.value == expected) {
   this.value = update;
   return true;
  }
}}
```

Universal for any number of threads

Practical Implications

- Any architecture that does not provide a universal primitive has inherent limitations
- You cannot avoid locking for concurrent data structures …

Older Architectures

- IBM 360
  - testAndSet (getAndSet)
- NYU UltraComputer
  - getAndAdd
- Neither universal
  - Except for 2 threads

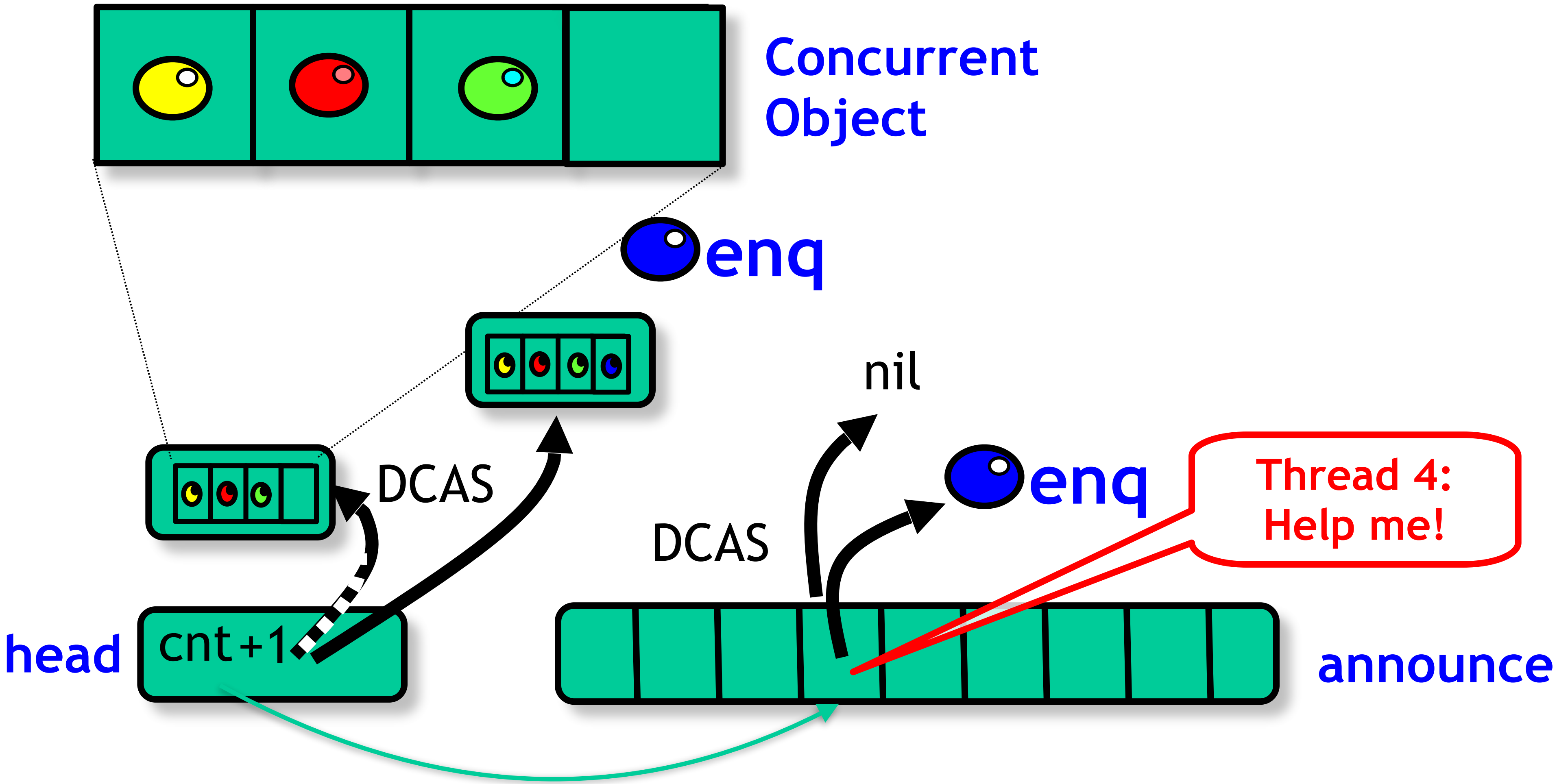
Newer Architectures

- Intel x86, Itanium, SPARC
  - compareAndSet
- Alpha AXP, PowerPC
  - Load-locked/store-conditional
- All universal
  - For any number of threads
- Trend is clear …

CAS-based and wait-free?

- Say, we only care about
  - compareAndSet (or, similar)
- We want to have wait-free, linearizable object
- Idea
  - reference and counter
  - counter says who to help
  - if nobody to help, i execute my method call

Wait-Free Implementation with DCAS



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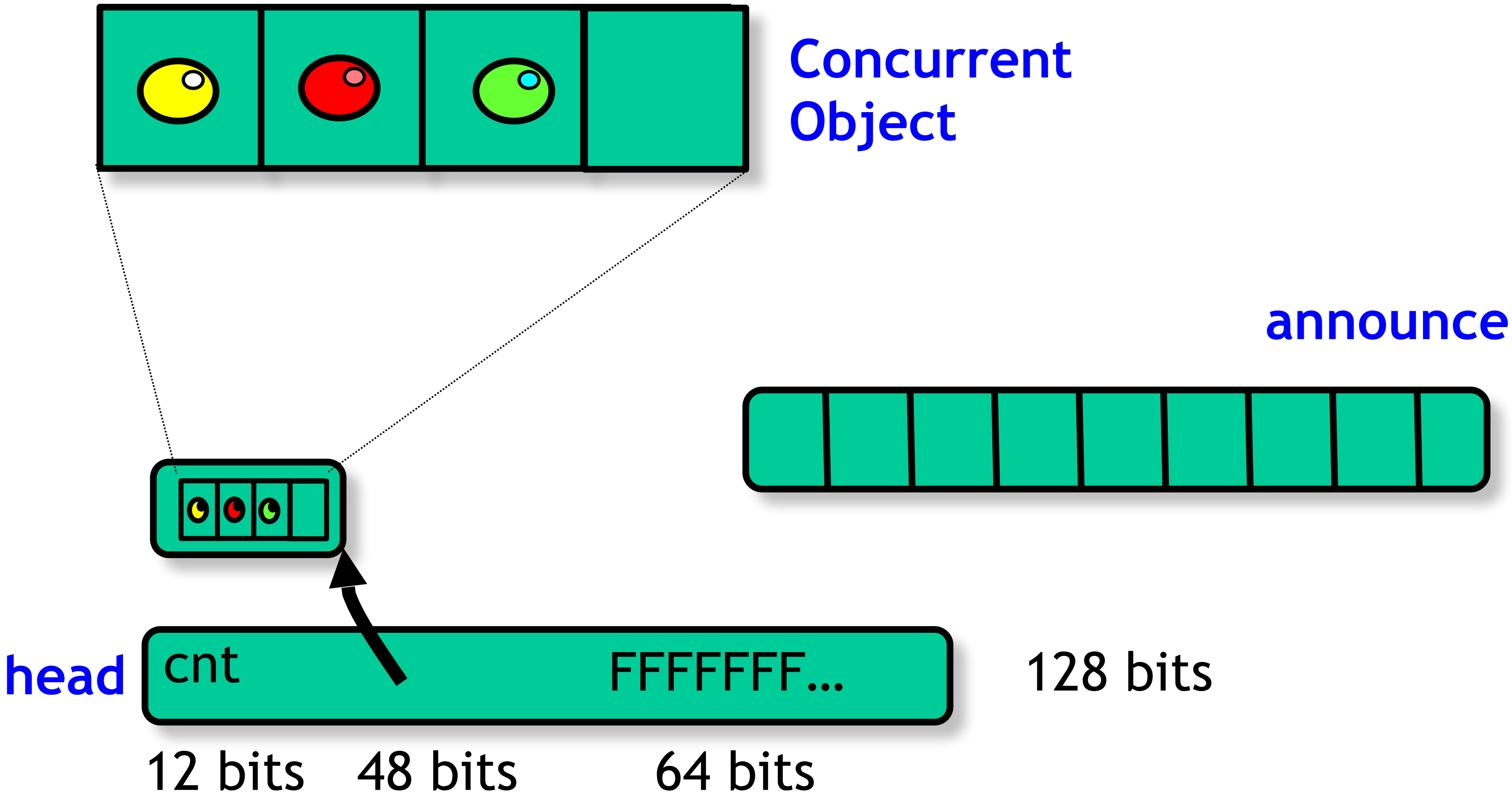
Alternative to DCAS

- We need two replace two distinct values
- Problem:
  - x86: can atomically replace 128bit value - but must be located in one cache line!
- If at most 64 threads:
  - use bit array to indicate that thread needs help (instead of reference)
  - set with „lock; or“ - guaranteed to succeed

Wait-Free Implementation with CMPXCHG16B



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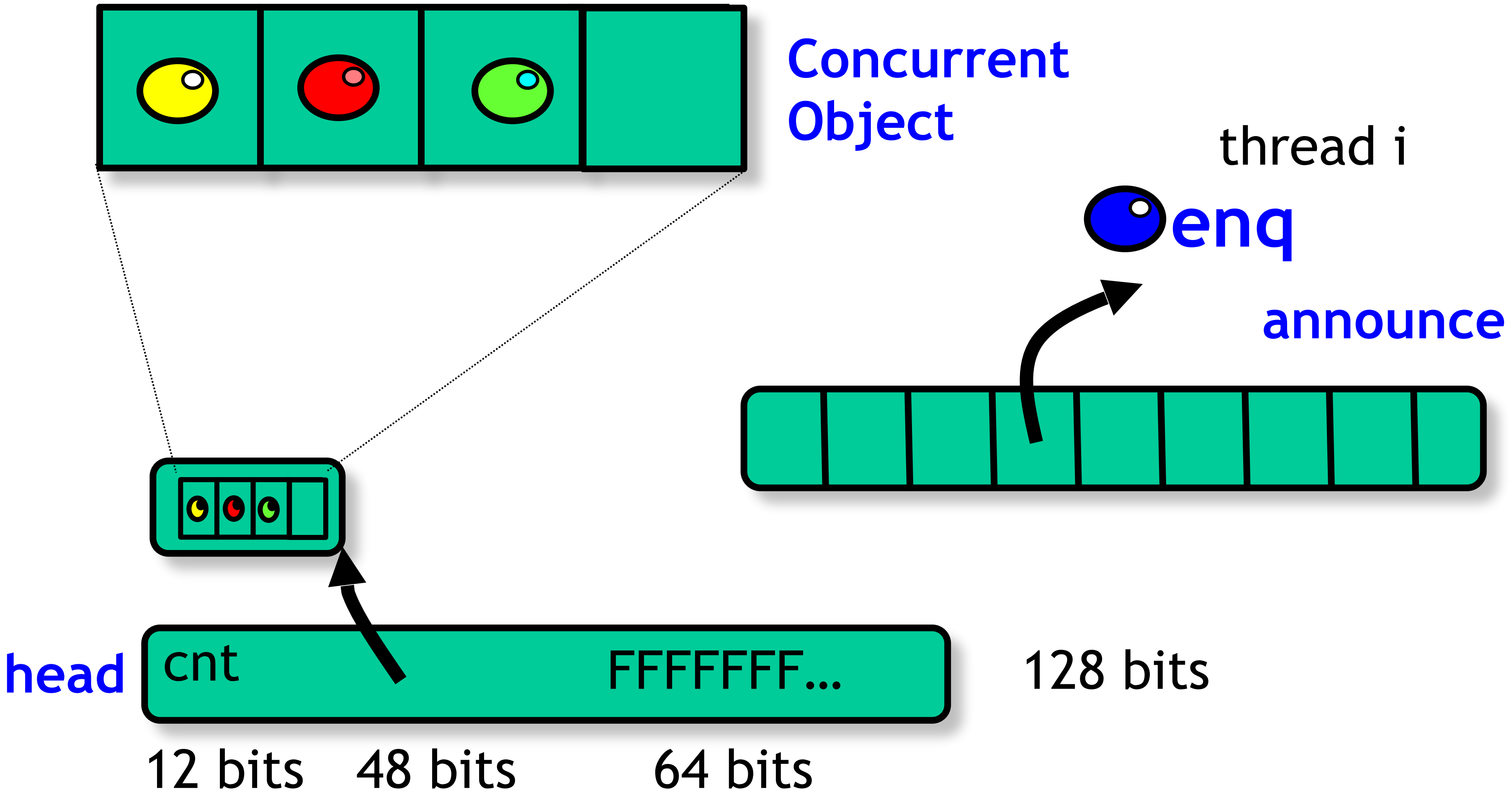
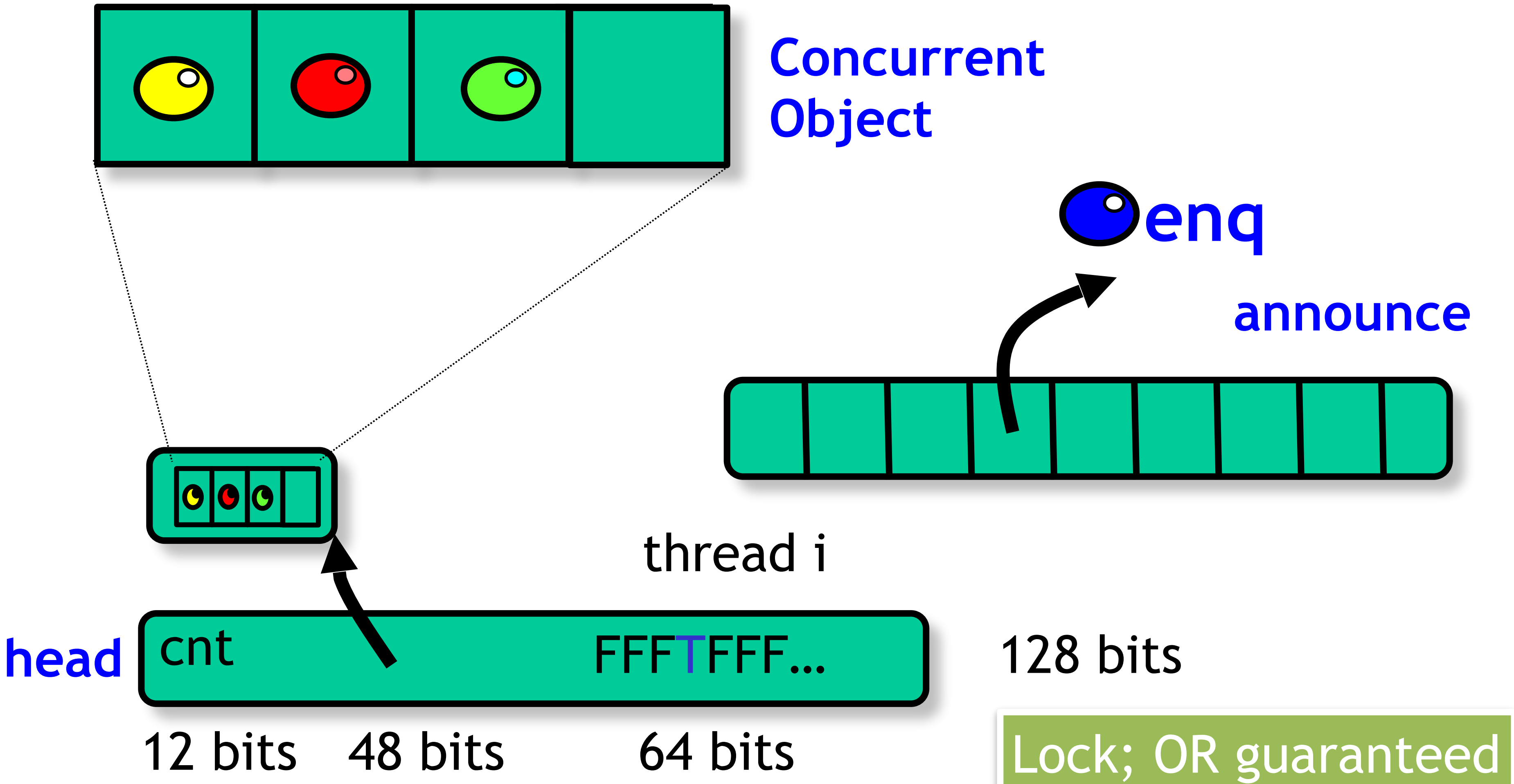
Wait-Free Implementation with CMPXCHG16B



„Lock; OR“



Lock; OR guaranteed to succeed!

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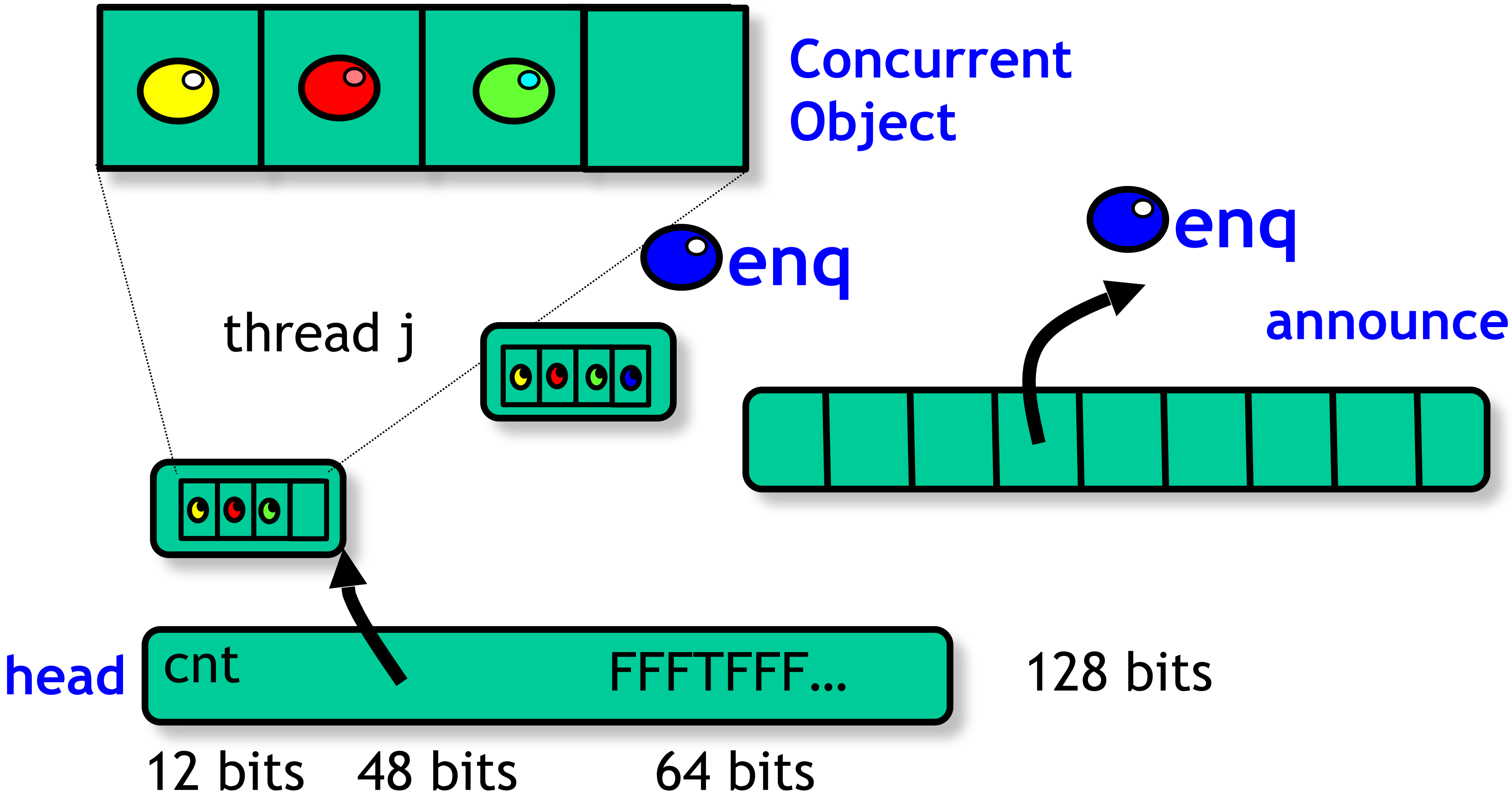
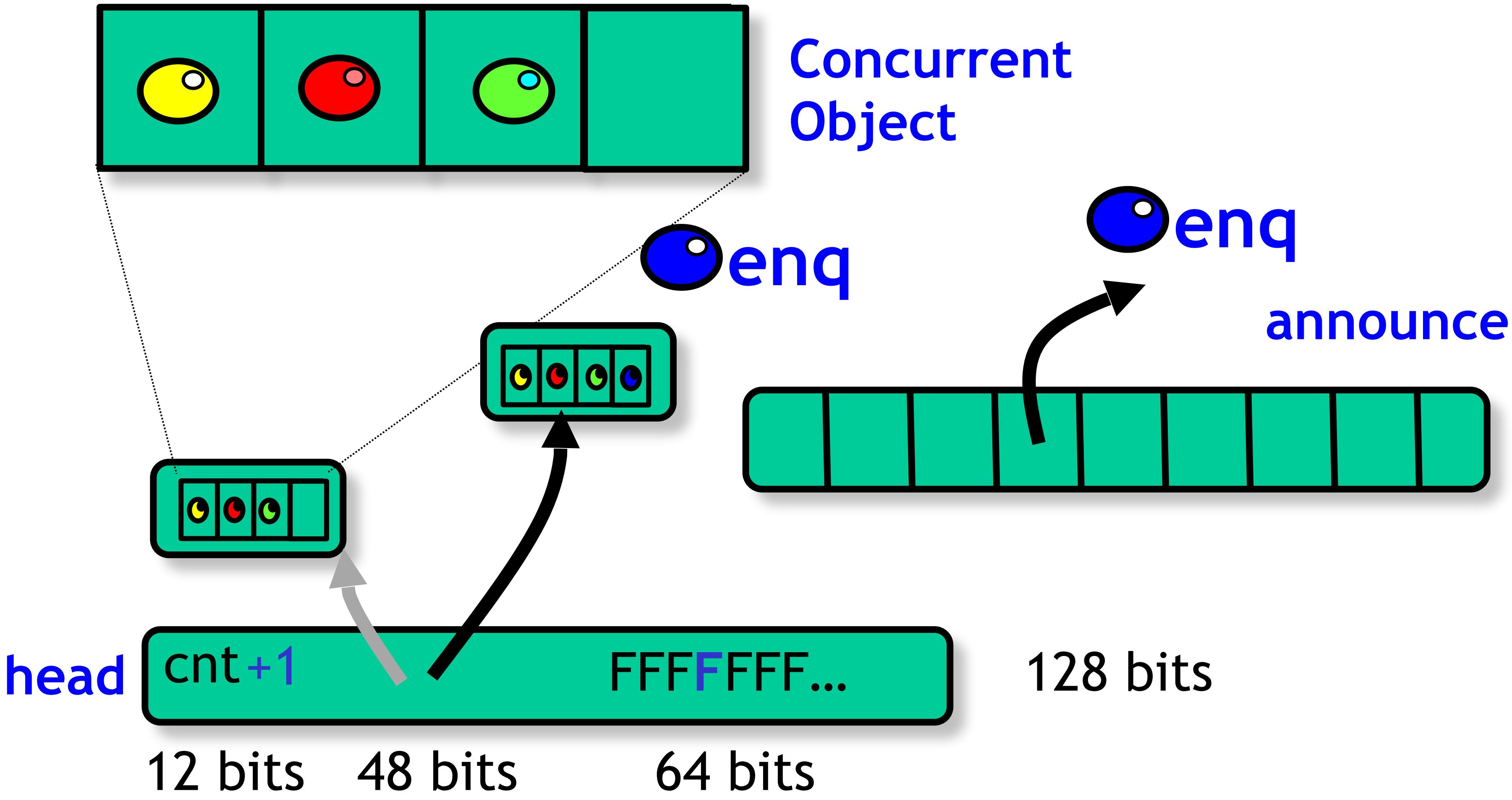
Wait-Free Implementation with CMPXCHG16B



CAS



Concurrent Object

enq

enq

announce

head

cnt+1

FFFFFFF...

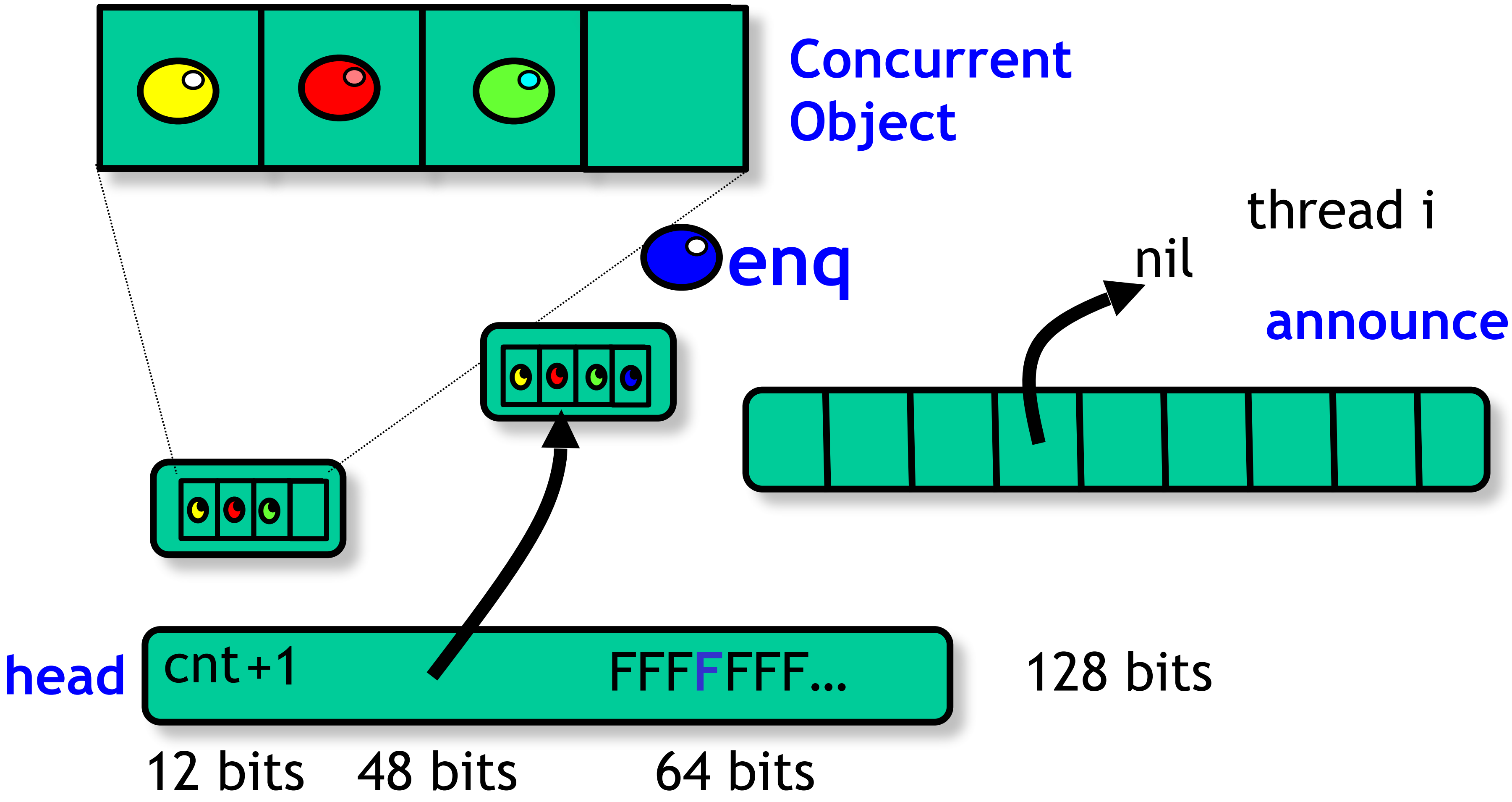
128 bits

12 bits

48 bits

64 bits

CAS



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Alternative to DCAS

- We need two replace two distinct values

- Alternative:
  - x86: transactional memory
  - however, only very weak progress guarantees

Weak Guarantees

- Transactional memory is obstruction-free
  - no interference: will succeed
  - interference: no guarantees
- Combine with exponential back off
  - back off to minimize interference