FOUNDATIONS OF CONCURRENT AND DISTRIBUTED SYSTEMS

- LINEARIZABILITY -

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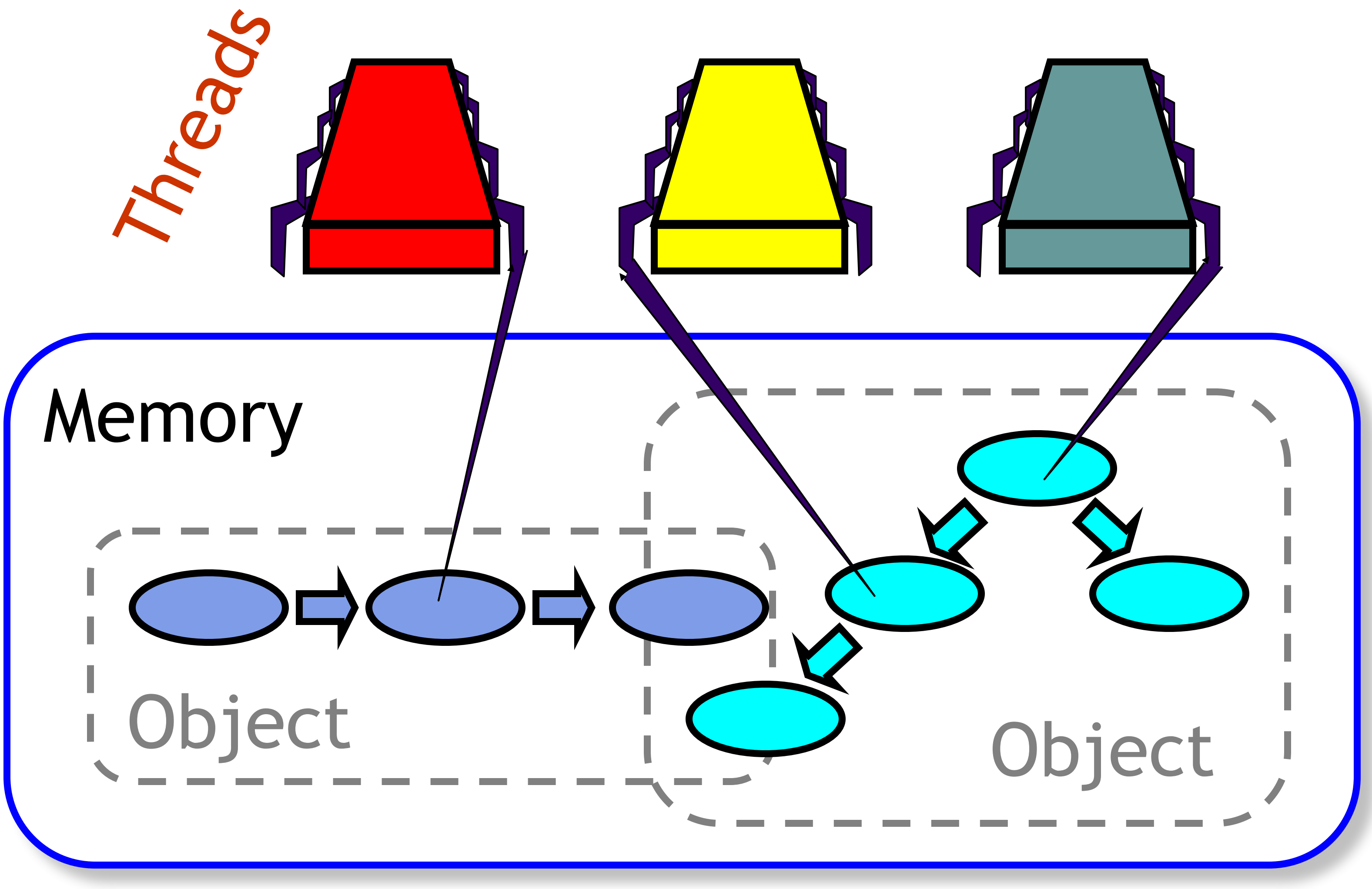
“Linearizability”

Concurrent Objects and Consistency

Prof. Christof Fetzer

(Based on the book and slides
The Art of Multiprocessor Programming
by Maurice Herlihy and Nir Shavit and Pascal Felber)

Concurrent Computation

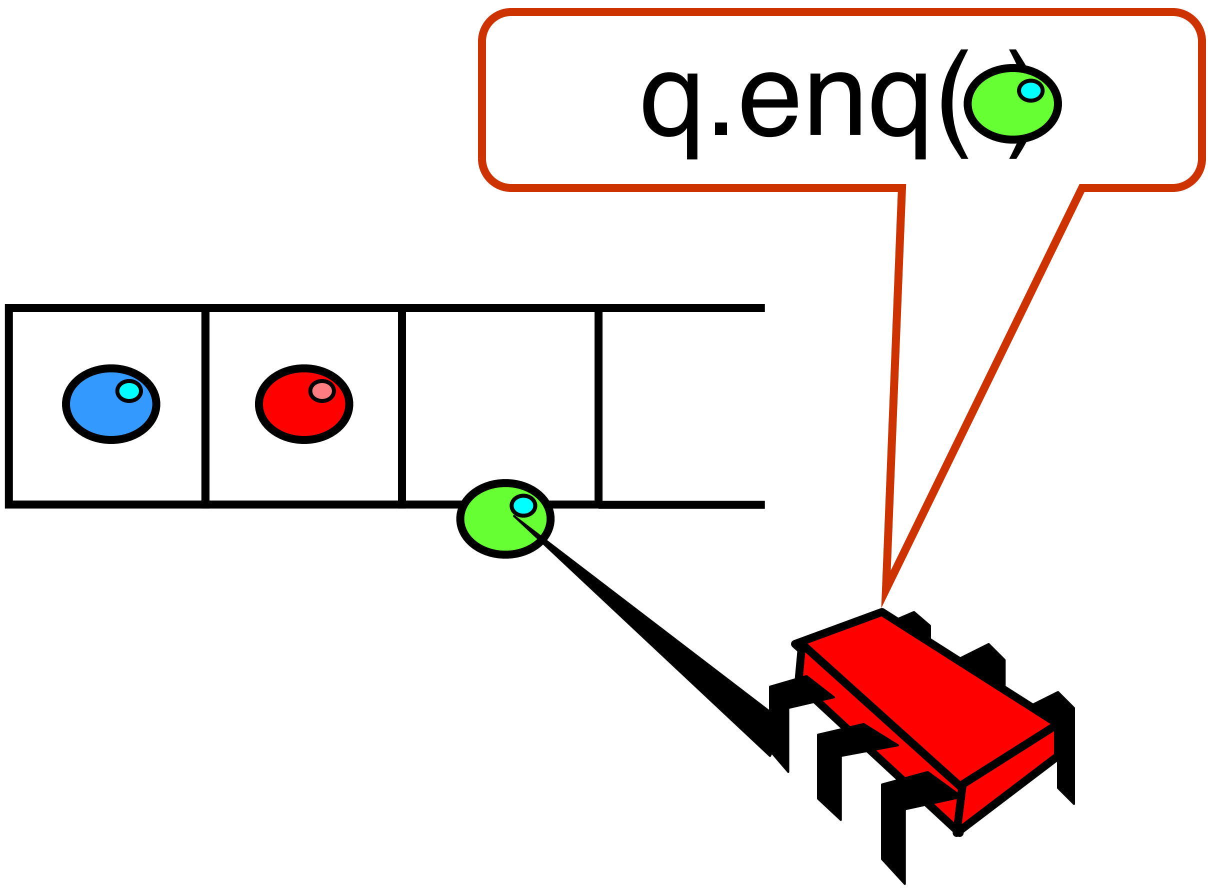


Objectivism

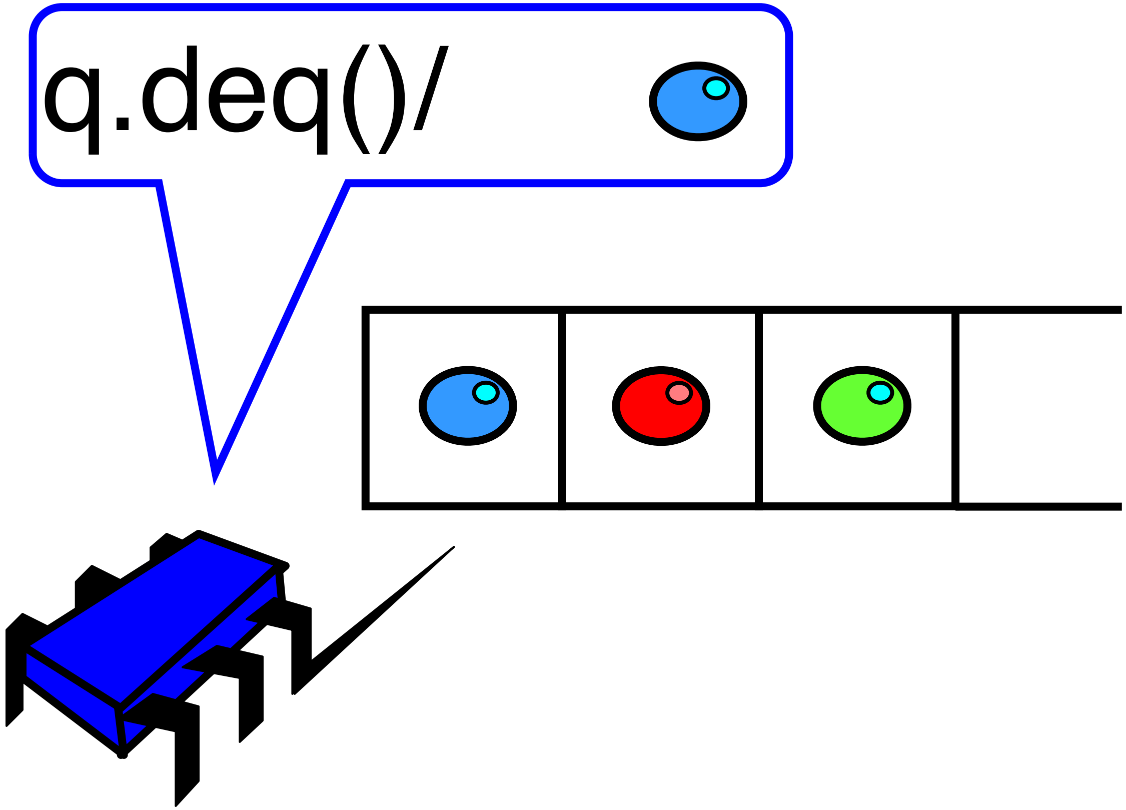
- What is a concurrent object?
  - How do we **describe** one?
  - How do we **implement** one?
  - How do we tell **tell if we're right**?

This lecture

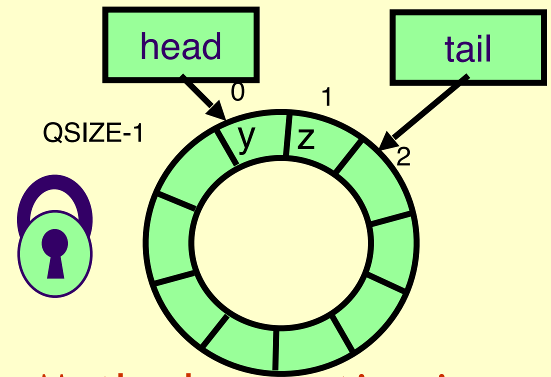
FIFO Queue: Enqueue Method



FIFO Queue: Dequeue Method



Implementation: enq()




```
public class Queue {
  int head = 0, tail = 0;
  Object[QSIZE] items;

  public synchronized
  void enq(Object x) {
    while (tail – head == QSIZE)
      this.wait();
    items[tail % QSIZE] = x;
    tail++;
    this.notifyAll();
  } …
}
```

Method execution is mutually exclusive

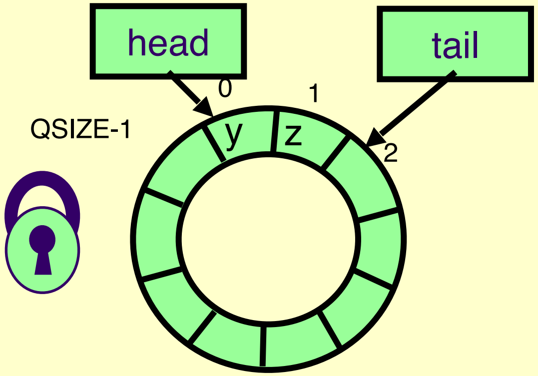
Release lock if need to wait

Here is where the queue is actually updated!

Notify all others that you release the lock

Implementation: deq()

```
public class Queue {
  int head = 0, tail = 0;
  Object[QSIZE] items;

  public synchronized
  Object deq() {
    while (tail – head == 0)
      this.wait();
    Object x = items[head % QSIZE];
    head++;
    this.notifyAll();
    return x;
  } …
}
```



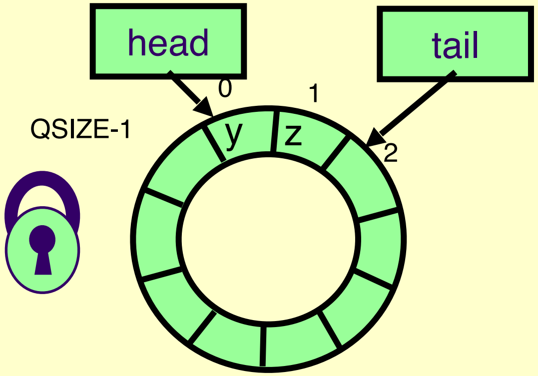
We understand it is “correct” because all modifications are mutually exclusive…

A Concurrent Implementation

- Now consider the following implementation
  - The same thing without mutual exclusion
- For simplicity, **only two threads**:
  - One thread only calls enq()
  - The other only deq()

Lock-free 2-Thread Queue

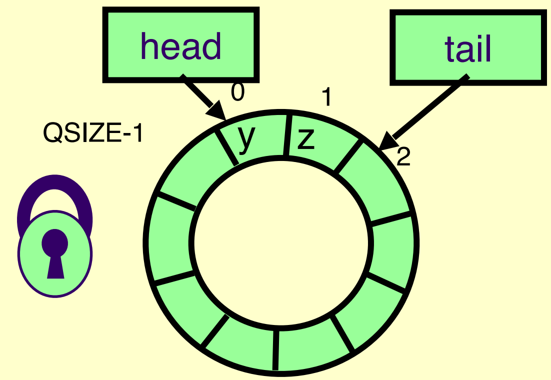
```
public class LockFreeQueue {
  volatile int head = 0,
               tail = 0;
  Object[QSIZE] items;
  public void enq(Item x) {
    while (tail-head == QSIZE); // busy-wait
    items[tail % QSIZE] = x; tail++;
  }
  public Item deq() {
    while (tail == head); // busy-wait
    Item item = items[head % QSIZE]; head++;
    return item;
  }
}
```



Queue updated without lock

How do we define "correct" when modifications are not exclusive?

Lock-free 2-Thread Queue




```
public class LockFreeQueue {
  volatile int head = 0,
               tail = 0;
  Object[QSIZE] items;
  public void enq(Item x) {
    if (tail-head == QSIZE)
            throw new Full();
    items[tail % QSIZE] = x; tail++;
  }
  public Item deq() {
    while (tail == head)
            throw new Empty();
    Item item = items[head % QSIZE]; head++;
    return item;
  }
}
```

How do we define “correct” when modifications are not exclusive?

Defining "Correct"

- Need a way to **specify** a concurrent queue object
- Need a way to prove that an algorithm **implements** the object's specification
- Lets talk about object specifications…

Sequential Objects

- Each object has a **state**
  - Usually given by a set of fields
  - Queue example: sequence of items
- Each object has a set of **methods**
  - Only way to manipulate state
  - Queue example: enq() and enq() methods

Sequential Specifications

- If (precondition)…
  - The object is in such-and-such a state…
  - Before you call the method…
- Then (postcondition)…
  - The method will return a particular value…
  - Or throw a particular exception…
- And (postcondition, con't)…
  - The object will be in some other state…
  - When the method returns

Pre- and Postconditions: deq()

- Precondition
  - Queue is non-empty
- Postcondition
  - Returns first item in queue
- Postcondition
  - Removes first item in queue

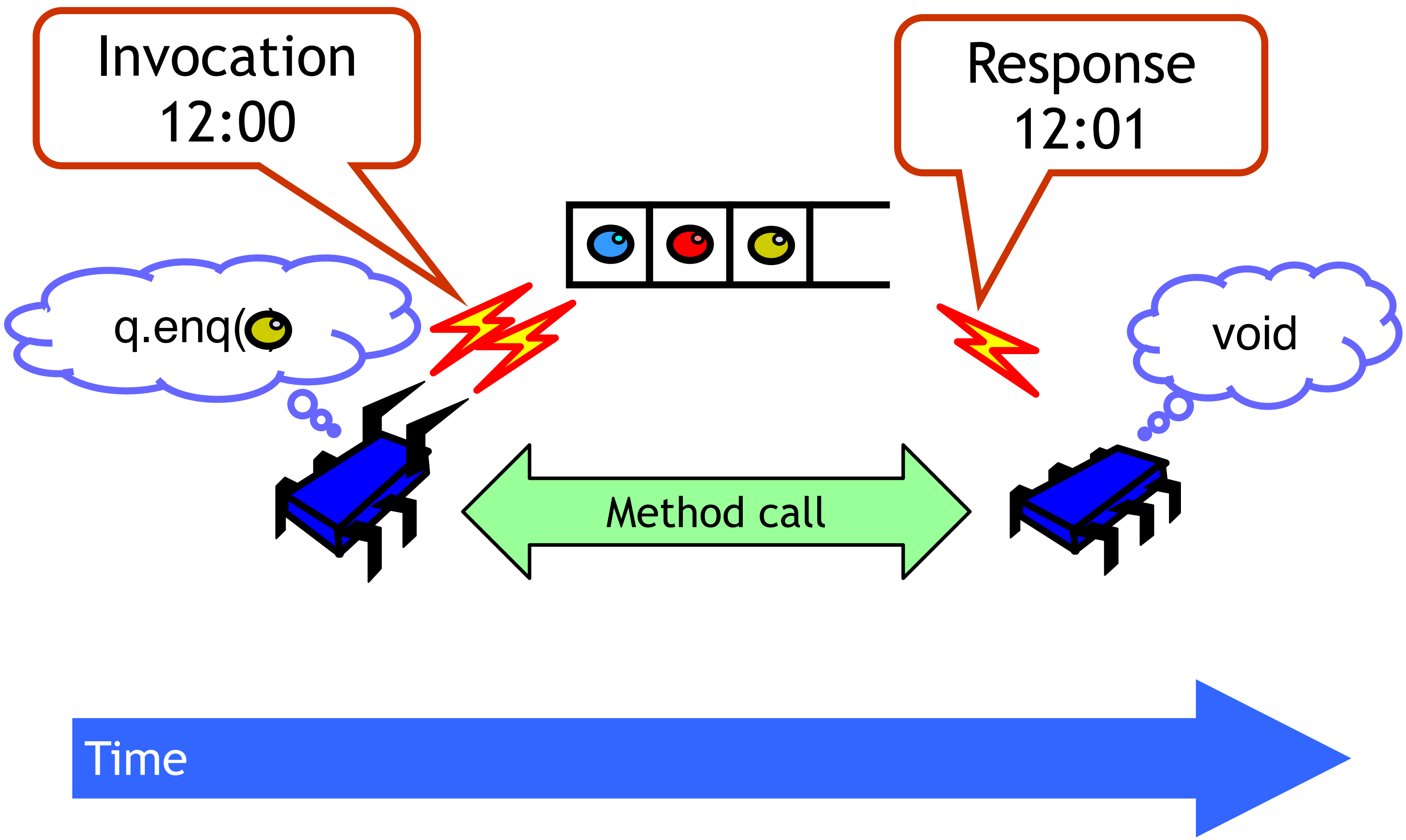
Pre- and Postconditions: deq()

- Precondition
  - Queue is empty
- Postcondition
  - Throws Empty exception
- Postcondition
  - Queue state unchanged

Why Sequential Specifications Totally Rock

- Interactions among methods captured by side-effects on object state
  - State meaningful between method calls
- Documentation size linear in number of methods
  - Each method described in isolation
- Can add new methods
  - Without changing descriptions of old methods
- What About Concurrent Specifications?
  - Methods? Documentation? Adding new methods?

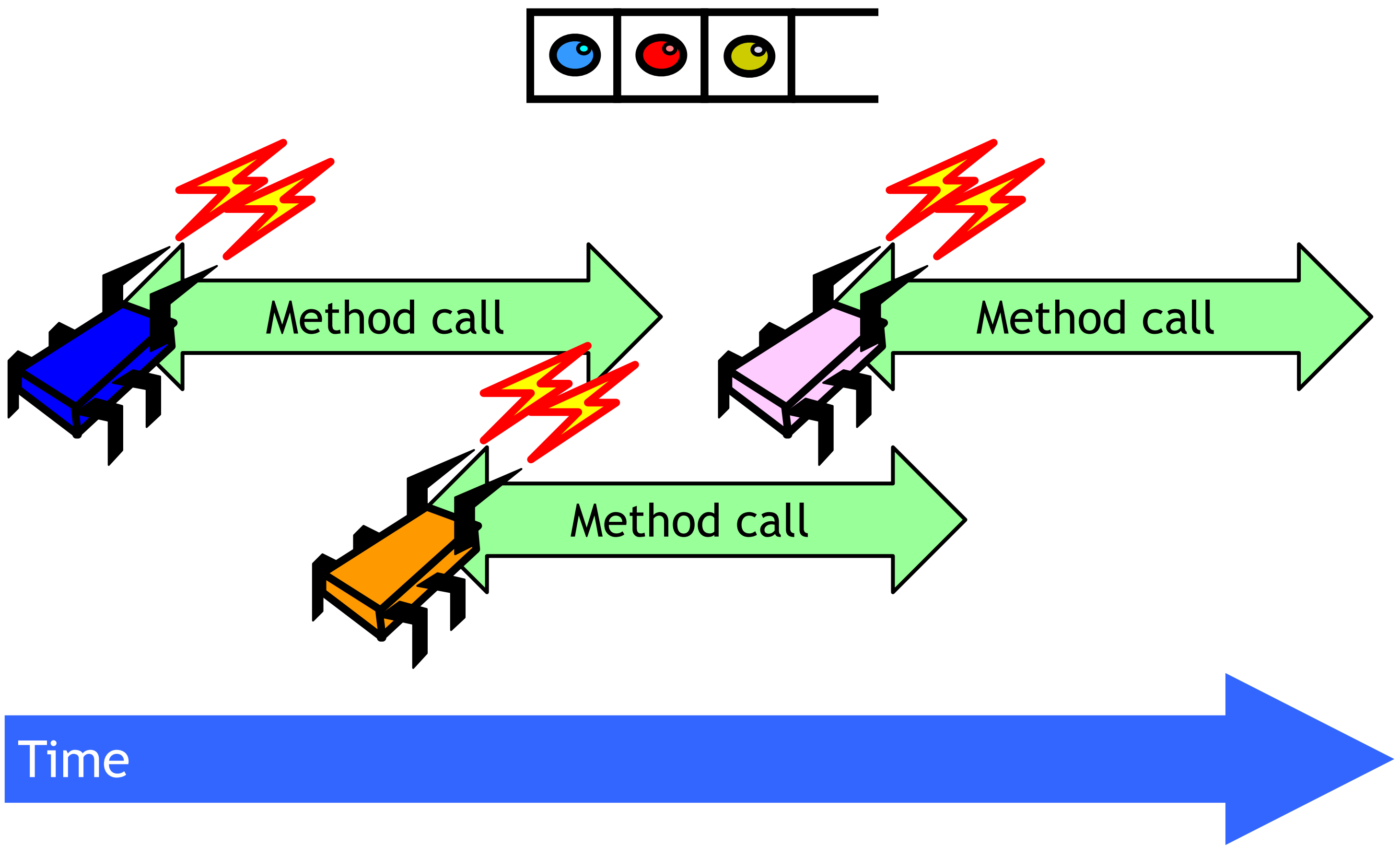
Methods Take Time



Sequential vs. Concurrent

- Sequential
  - Methods take time? Who knew?
- Concurrent
  - Method call is not an event
  - Method call is an interval

Concurrent Methods Take Overlapping Time



Sequential vs. Concurrent

Sequential

- Object needs meaningful state only between method calls
- Each method described in isolation
- Can add new methods without affecting older methods

Concurrent

- Because method calls overlap, object might never be between method calls
- Must characterize all possible interactions with concurrent calls
  - What if 2 enq() overlap? Or 2 deq()? Or enq() and deq()?
- Everything can potentially interact with everything else

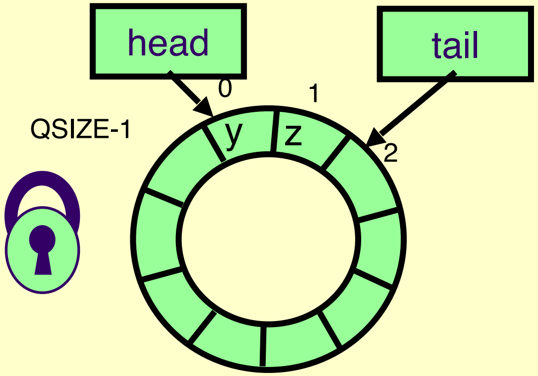
PANIC!

The Big Question

- What does it mean for a concurrent object to be correct?
  - What is a concurrent FIFO queue?
  - FIFO means strict temporal order
  - Concurrent means ambiguous temporal order

Intuitively…

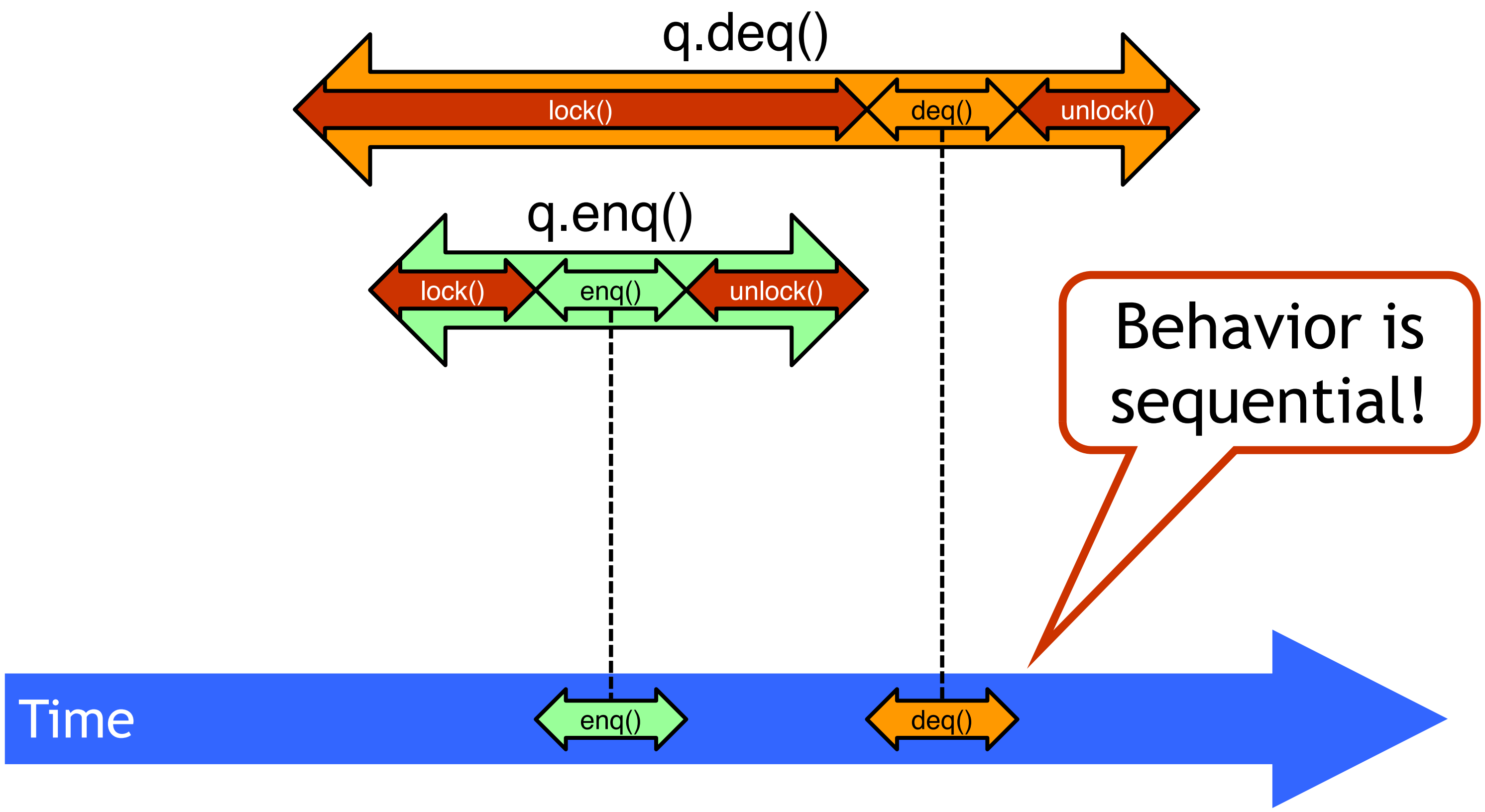
```
public class Queue {
  int head = 0, tail = 0;
  Object[QSIZE] items;

  public synchronized
  void enq(Object x) {
    while (tail – head == QSIZE)
      this.wait();
    items[tail % QSIZE] = x;
    tail++;
    this.notifyAll();
  } …
}
```



Queue is updated while holding lock (mutually exclusive)

Intuitively…

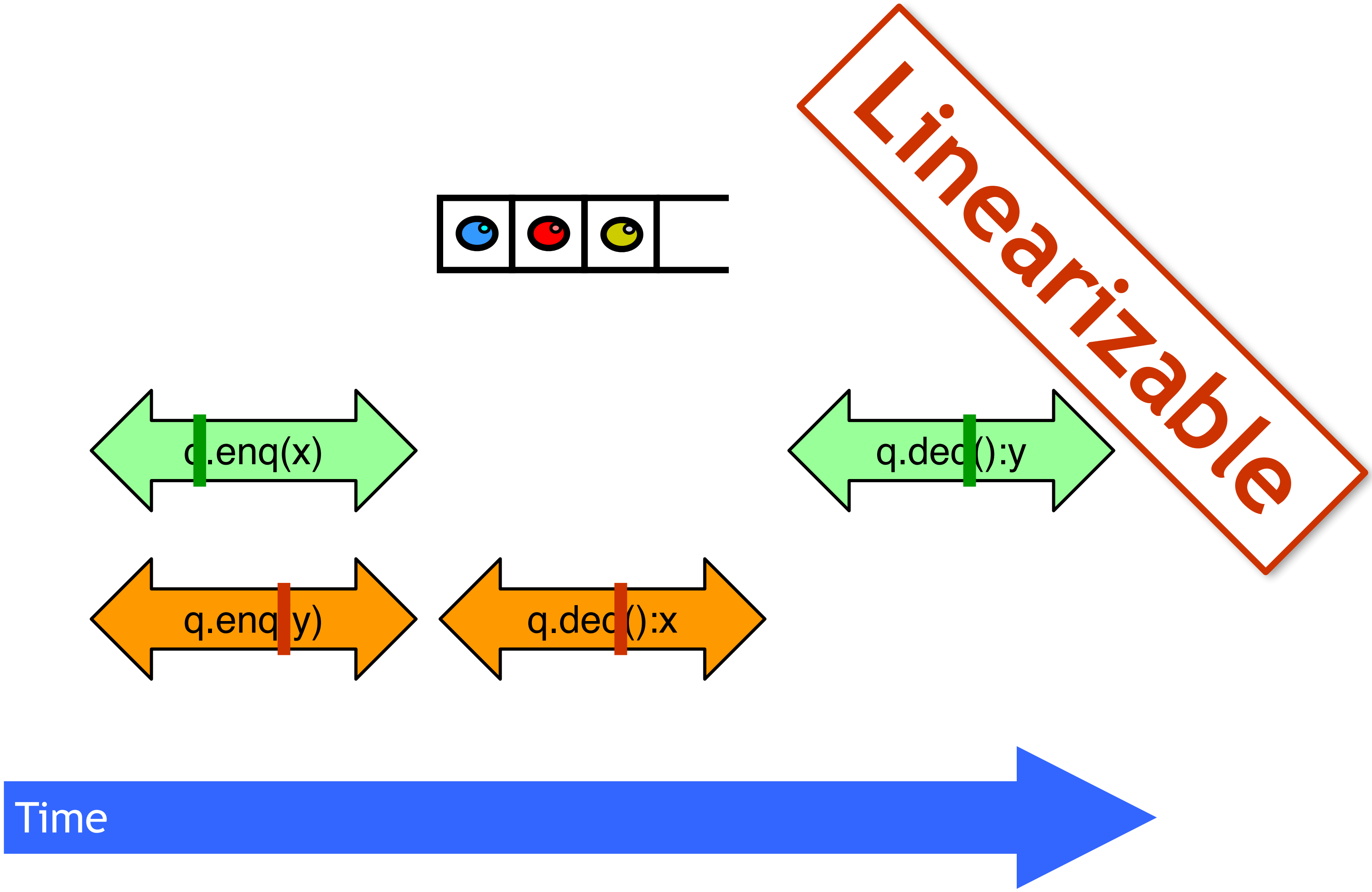
Lets capture the idea of describing the concurrent via the sequential



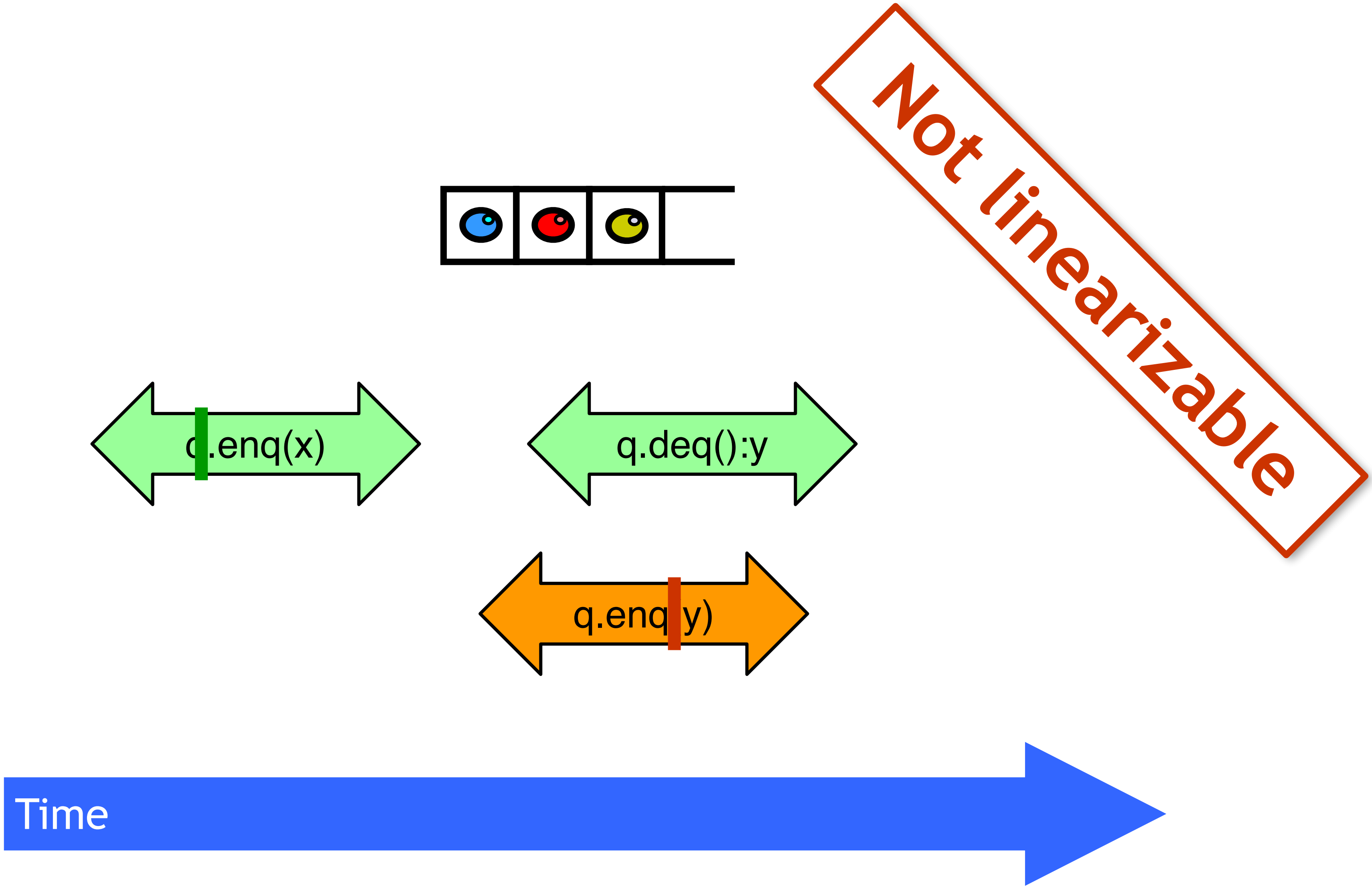
Linearizability

- Each method should
  - "Take effect"
  - Instantaneously
  - Between invocation and response events
- Object is correct if this "sequential" behavior is correct
- Any such concurrent object is **linearizable**
  - Formally, a **linearizable object** is an object all of whose possible executions are linearizable

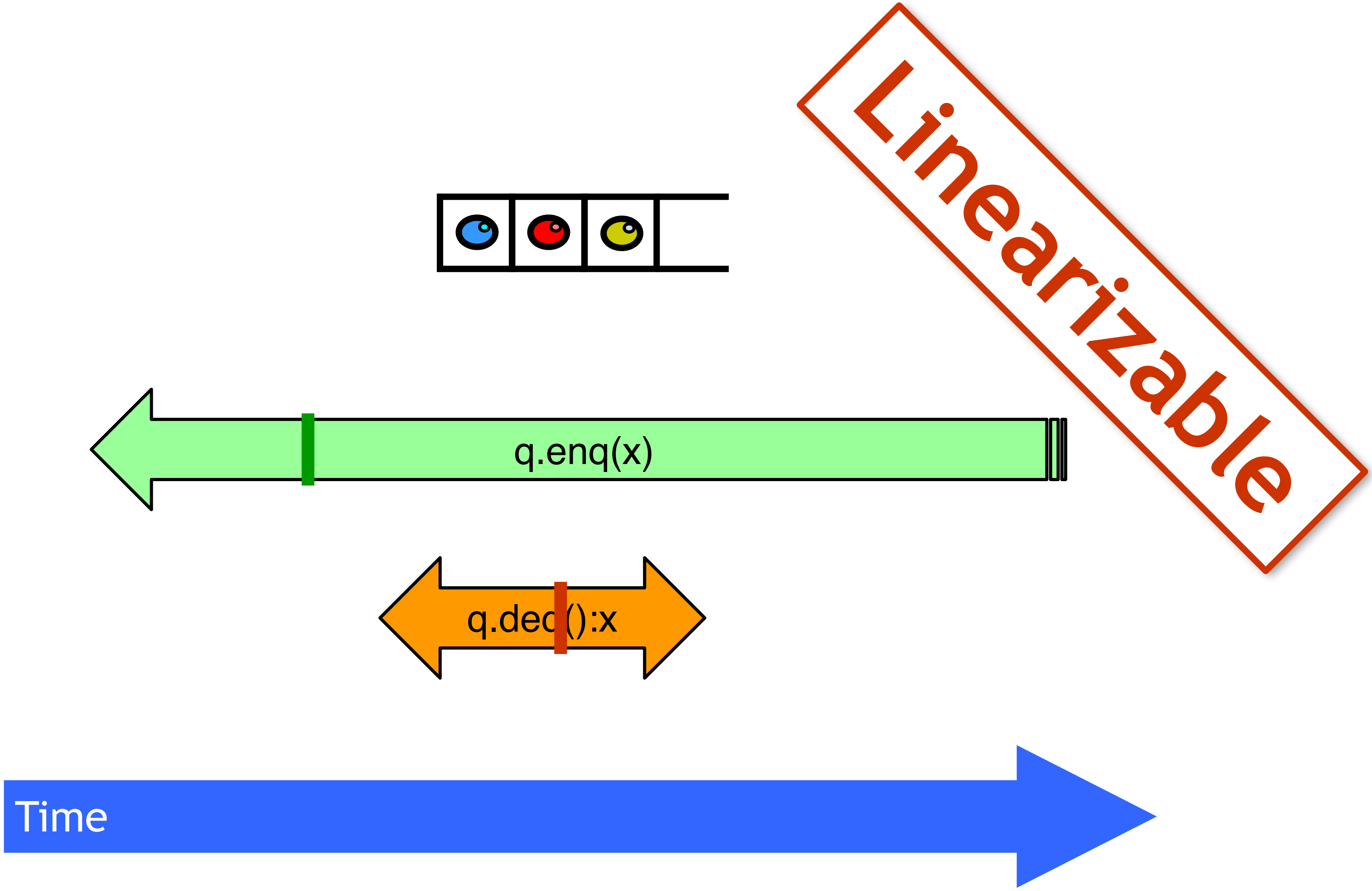
FIFO Queue Example



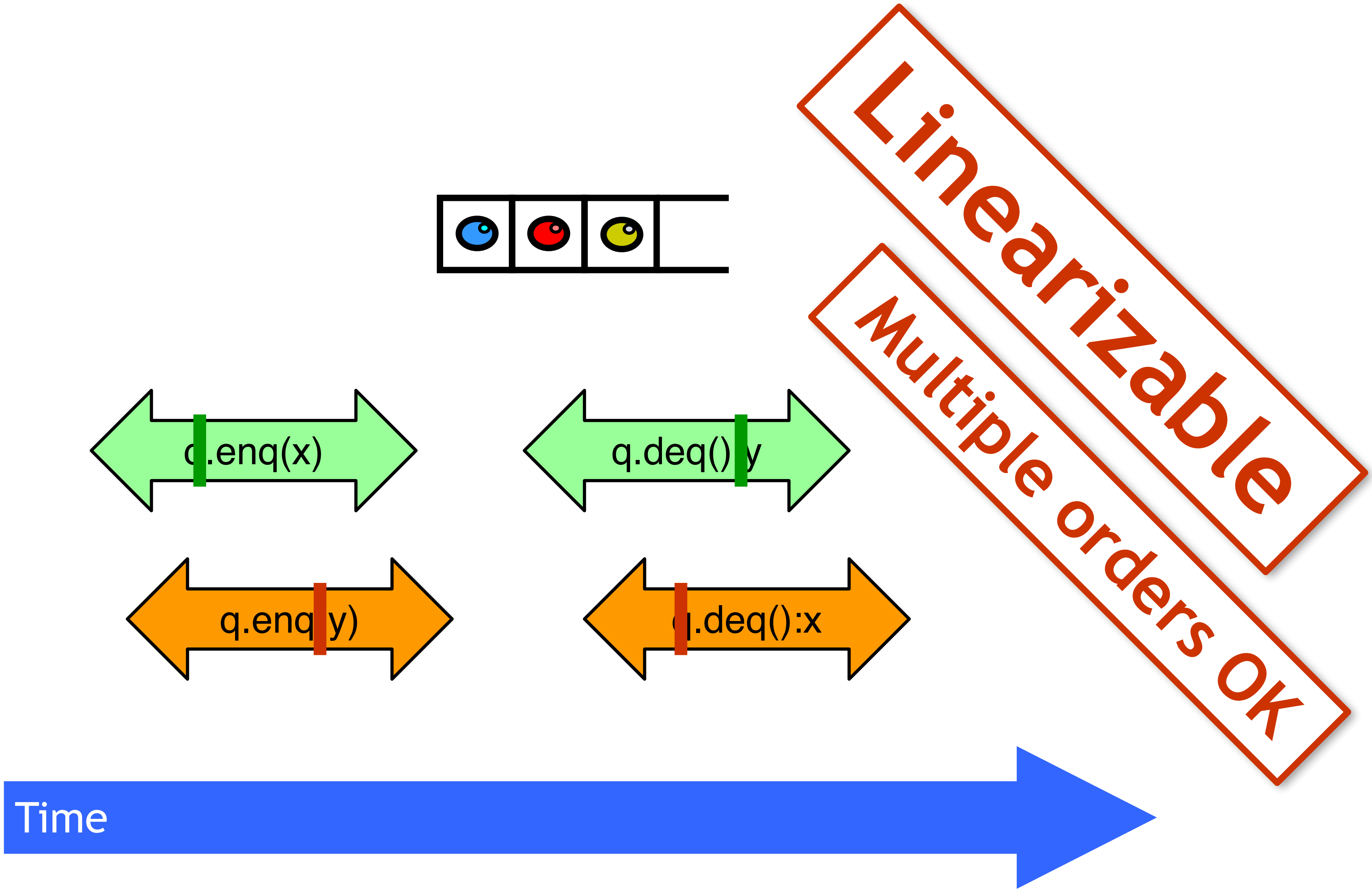
FIFO Queue Example



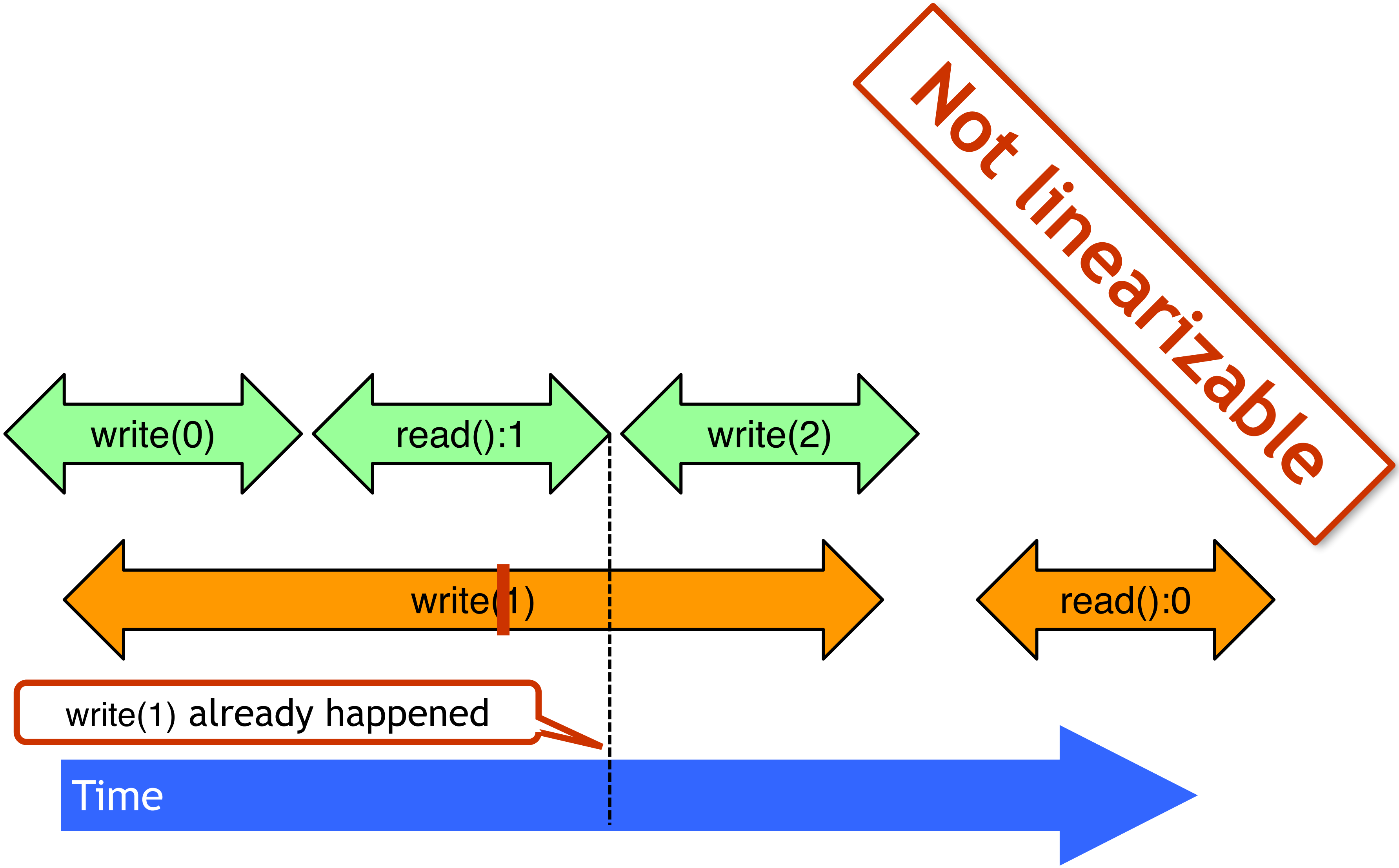
FIFO Queue Example



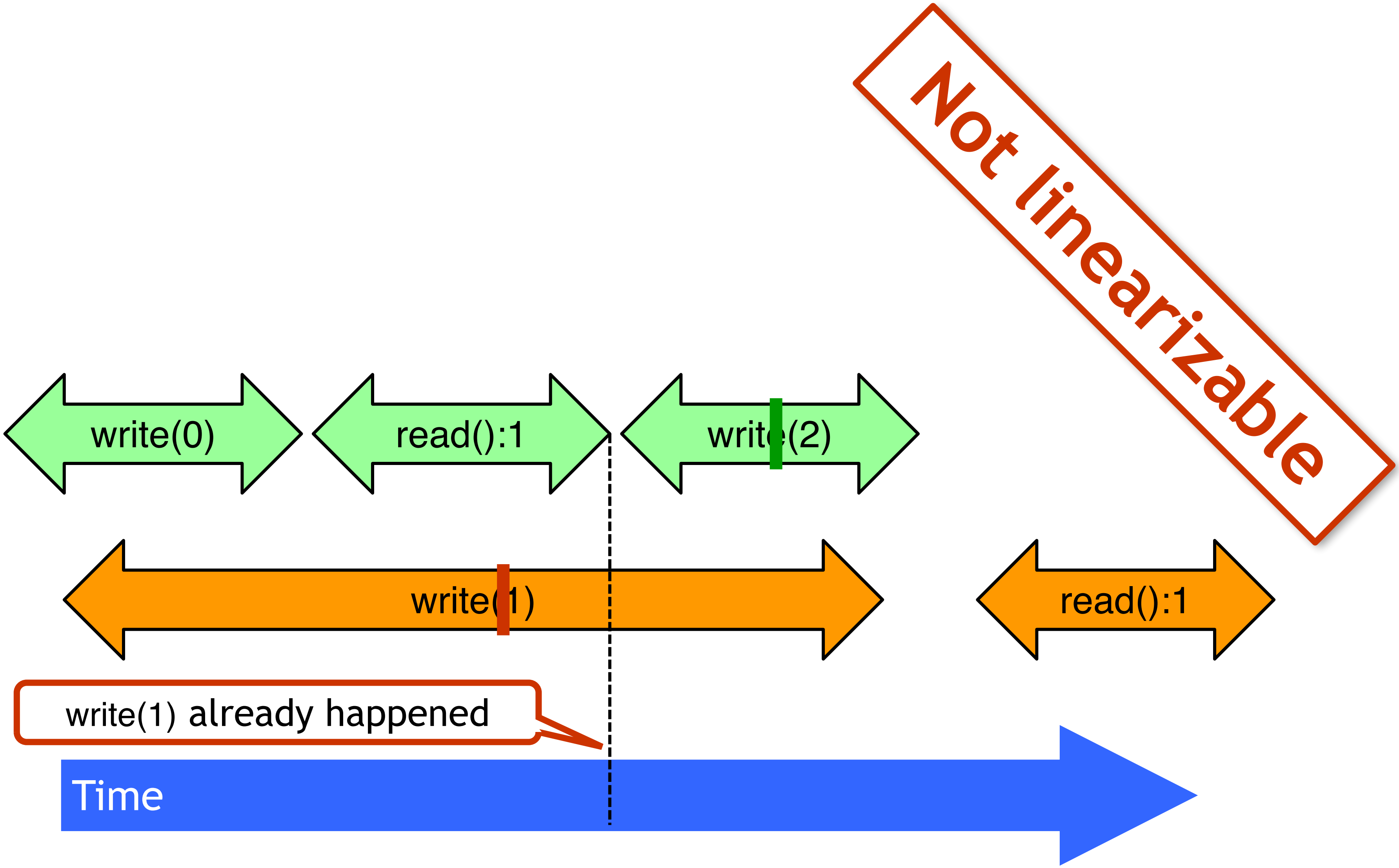
FIFO Queue Example



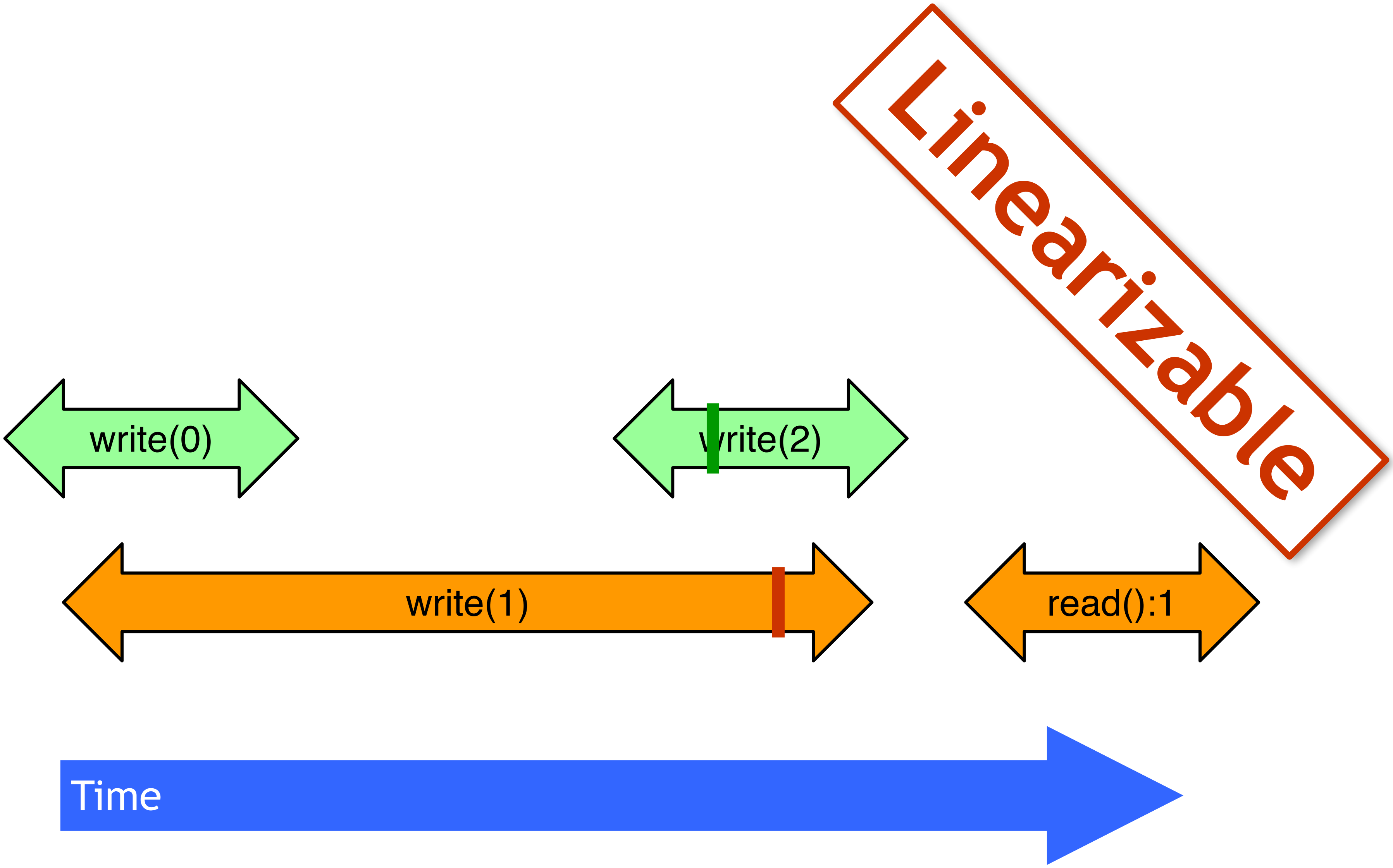
Read/Write Register Example



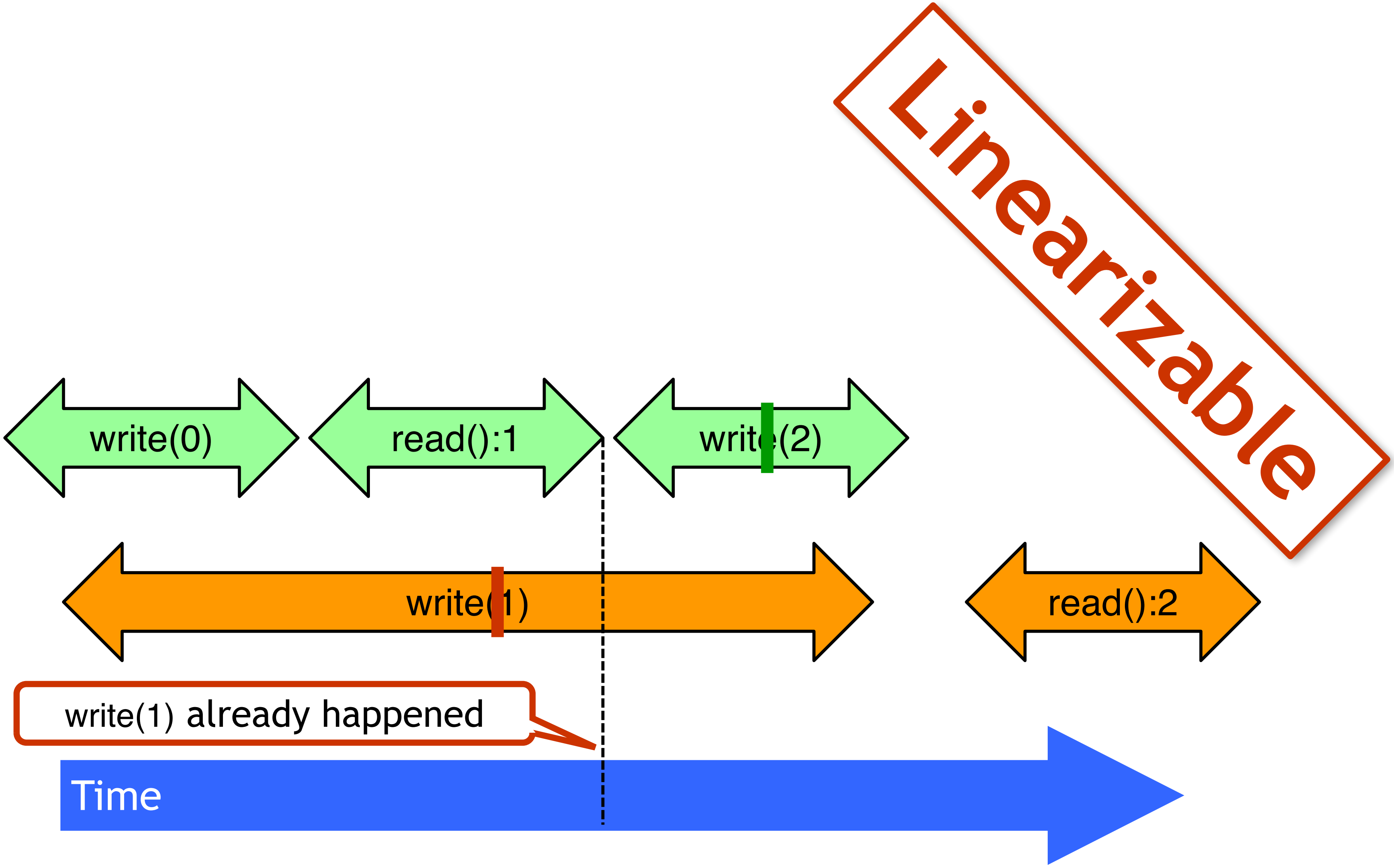
Read/Write Register Example



Read/Write Register Example



Read/Write Register Example



Talking About Executions

- Why do we need to consider executions?
  - Can't we specify the linearization point of each operation without describing an execution?
- Not Always
  - In some cases, linearization point depends on the execution
- Let's define a formal model of executions
  - Define precisely what we mean (ambiguity is bad)
  - Allow reasoning, formal or informal

Formal Definition: Split Method Calls into Two Events

- Invocation
  - Method name & arguments

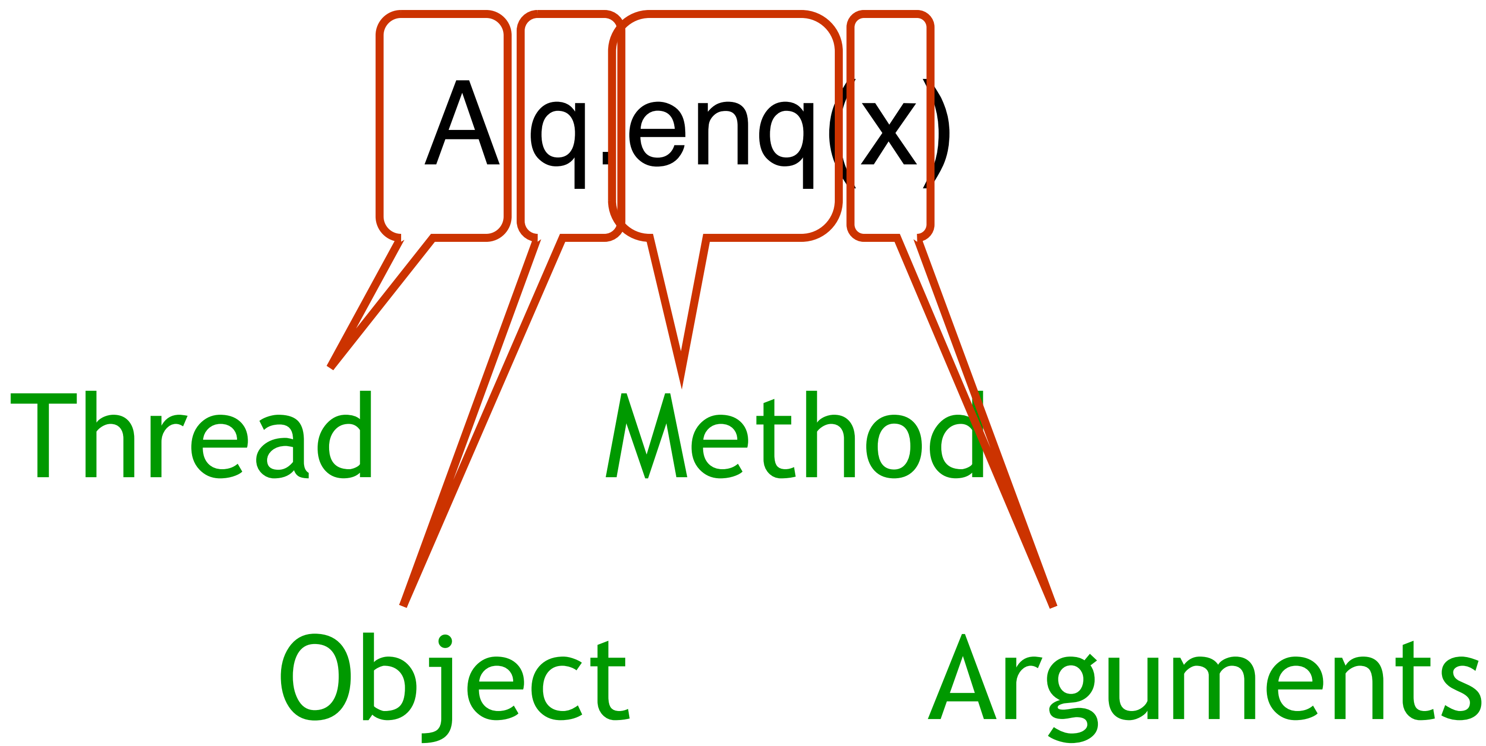
    q.enq(x)
- Response
  - Result or exception

    q.enq(x) returns void

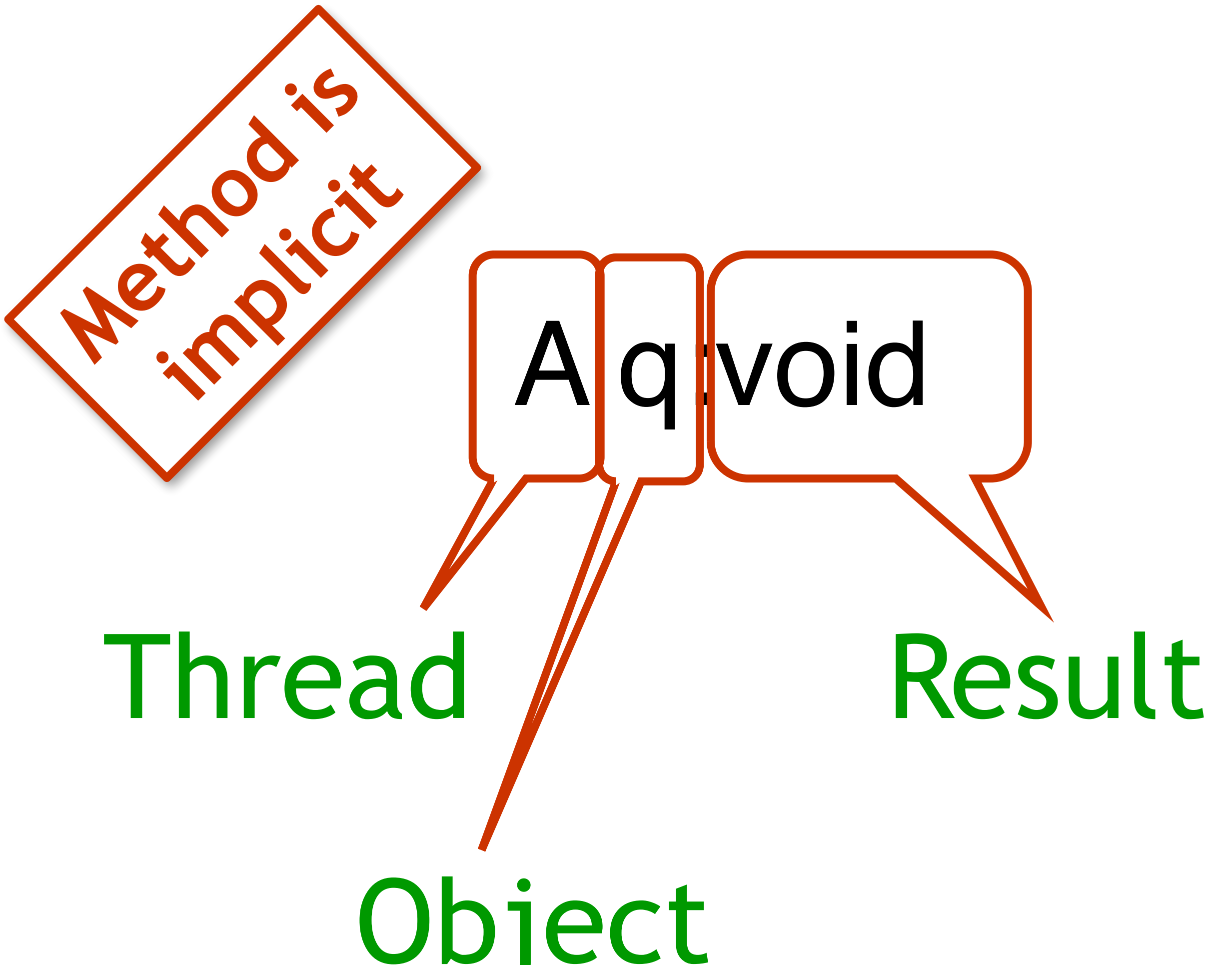
    q.deq() returns x

    q.deq() throws Empty

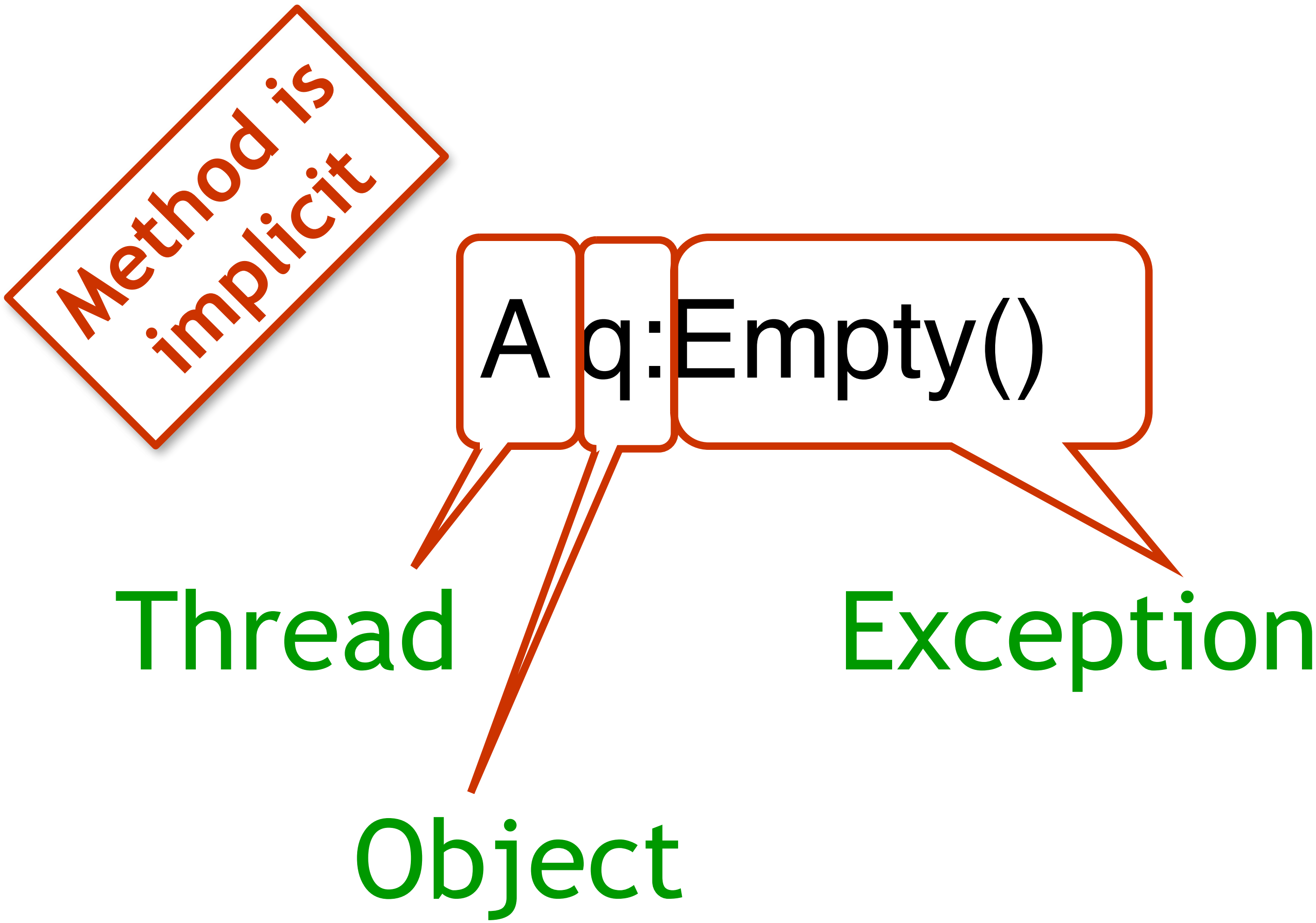
Invocation Notation



Response Notation



Response Notation



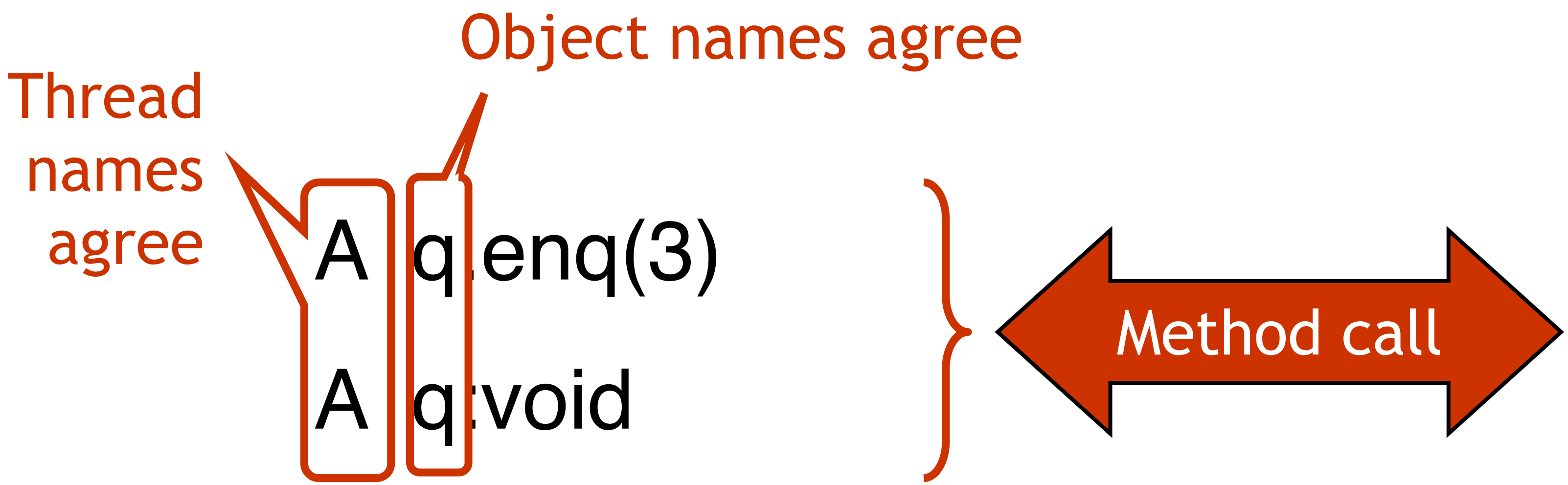
History: Describing an Execution

H =
A q.enq(3)
A q:void
A q.enq(5)
B p.enq(4)
B p:void
B q.deq()
B q:3

Sequence of invocations and responses

Definition

- Invocation & response match if



Object Projections

H =

A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3

H|q =

A q.enq(3)
A q:void

B q.deq()
B q:3

Thread Projections

H =

A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3

H|B =

B p.enq(4)
B p:void
B q.deq()
B q:3

Complete Subhistory

H = A q.enq(3)
A q:void
A q.enq(5)
B p.enq(4)
B p:void
B q.deq()
B q:3

An invocation is pending if it has no matching response

May or may not have taken effect ⇒ discard pending

invocations

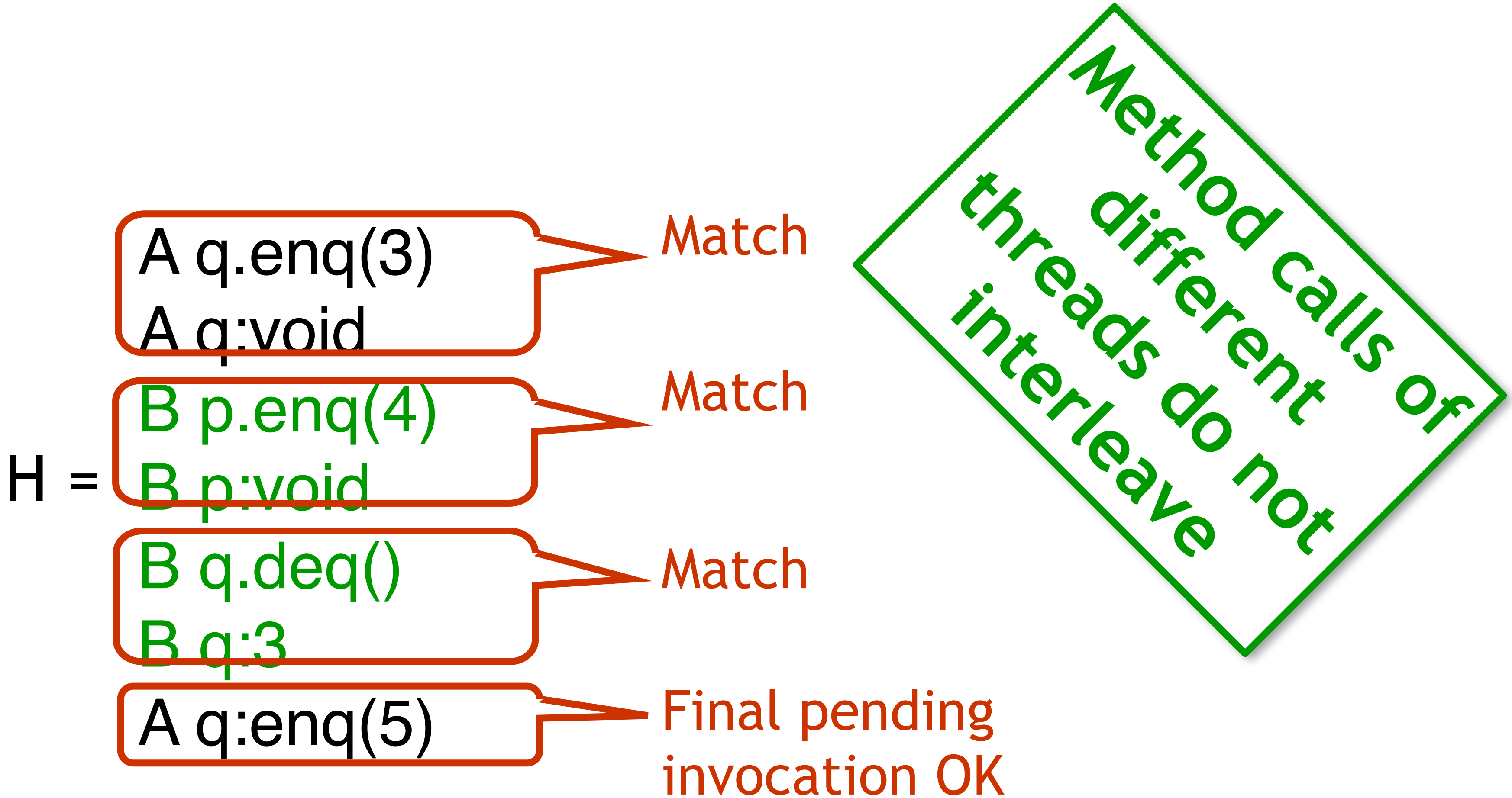
Complete Subhistory

A q.enq(3)
A q:void

Complete(H) =

B p.enq(4)
B p:void
B q.deq()
B q:3

Sequential Histories



Well-Formed Histories

Per-thread projections are sequential

H =
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3

H|B =
B p.enq(4)
B p:void
B q.deq()
B q:3

H|A =
A q.enq(3)
A q:void

Equivalent Histories

Threads see the same thing in both histories

$$\begin{cases} H|A = G|A \\ H|B = G|B \end{cases}$$

H =
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3

G =
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3

Sequential Specifications

- A sequential specification is some way of telling whether a…
  - Single-thread, single-object history…
  - Is legal
- Simple way is using…
  - Pre and post-conditions…
  - But plenty of other techniques exist

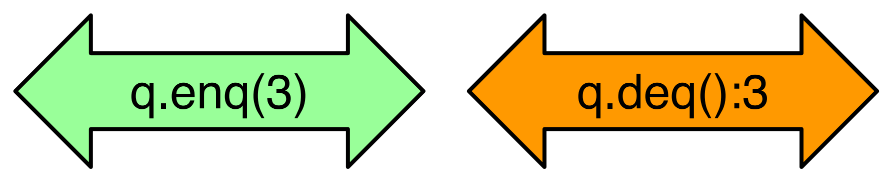
Legal Histories

- A sequential (multi-object) history H is legal if
  - For every object x
  - $H|x$ is in the sequential specification for x

Precedence

H = A q.enq(3)
B p.enq(4)
B p:void
A q:void
B q.deq()
B q:3

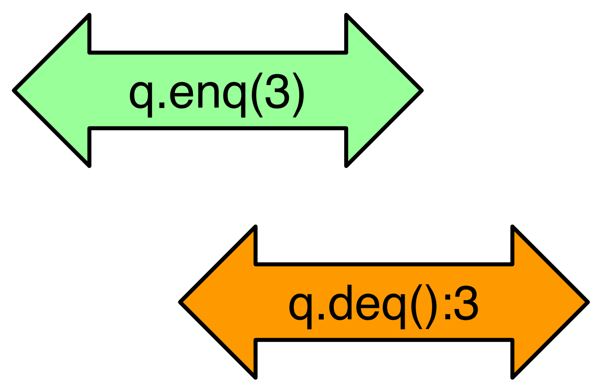
A method call precedes another if response event precedes invocation event



Non-Precedence

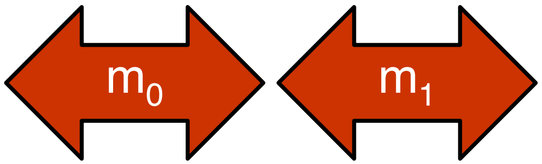
Some method calls *overlap* one another

H =
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3



Notation

- Given
  - History H
  - Method executions m_0 and m_1 in H
- We say $m_0 \rightarrow_H m_1$, if
  - m_0 precedes m_1



- Relation $\rightarrow_H$ is a
  - Partial order
  - Total order if H is sequential

Linearizability

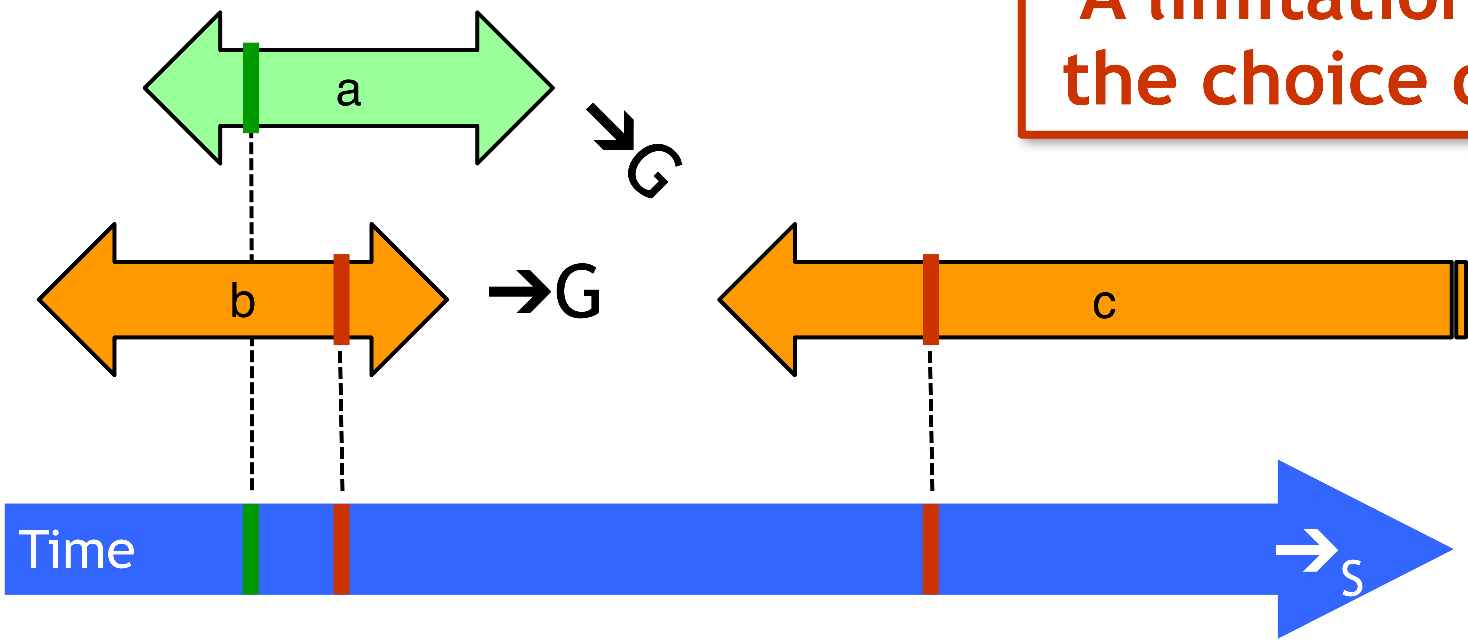
- History **H** is **linearizable** if it can be extended to **G** by
  - Appending zero or more responses to pending invocations
  - Discarding other pending invocations
- So that **G** is equivalent to
  - Legal sequential history **S**
  - Where $\rightarrow_G \subset \rightarrow_S$

What is →G ⊂ →S?

$\rightarrow_G$ = {a→c, b→c}

$\rightarrow_S$ = {a→b, a→c, b→c}

A limitation on the choice of S!



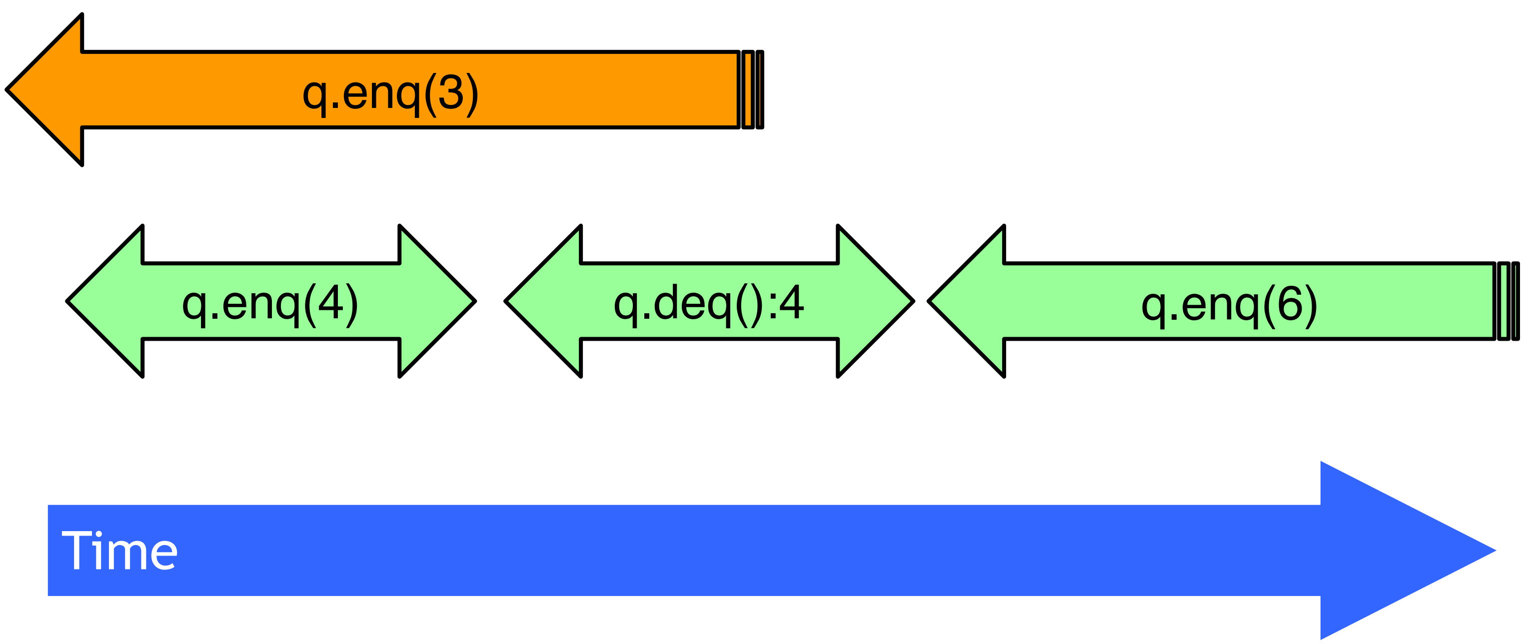
Remarks

- Some pending invocations
  - Took effect, so keep them
  - Discard the rest
- Condition $\rightarrow_G \subset \rightarrow_S$
  - Means that S respects "real-time order" of G

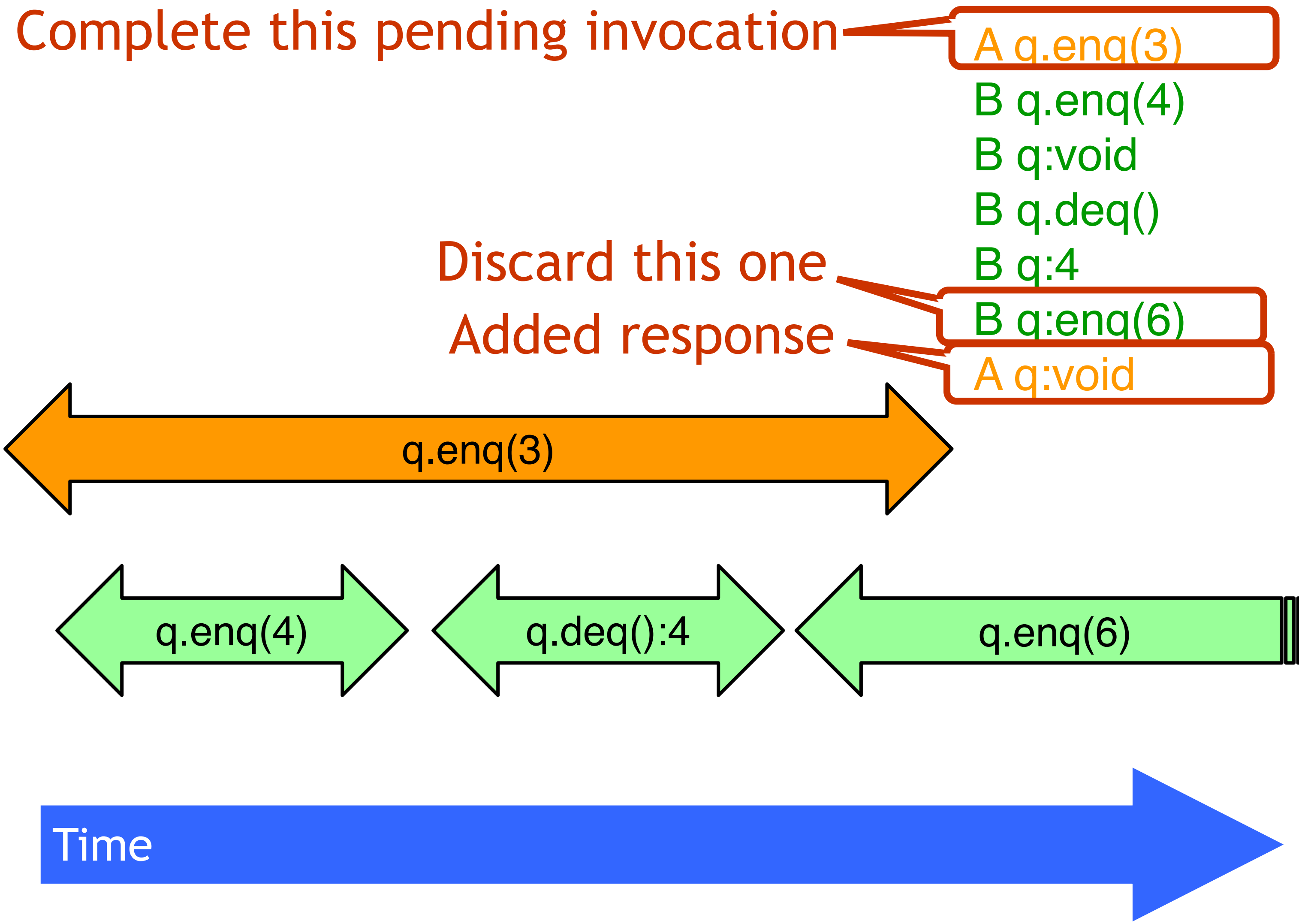
Example

Complete this pending invocation

A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)



Example

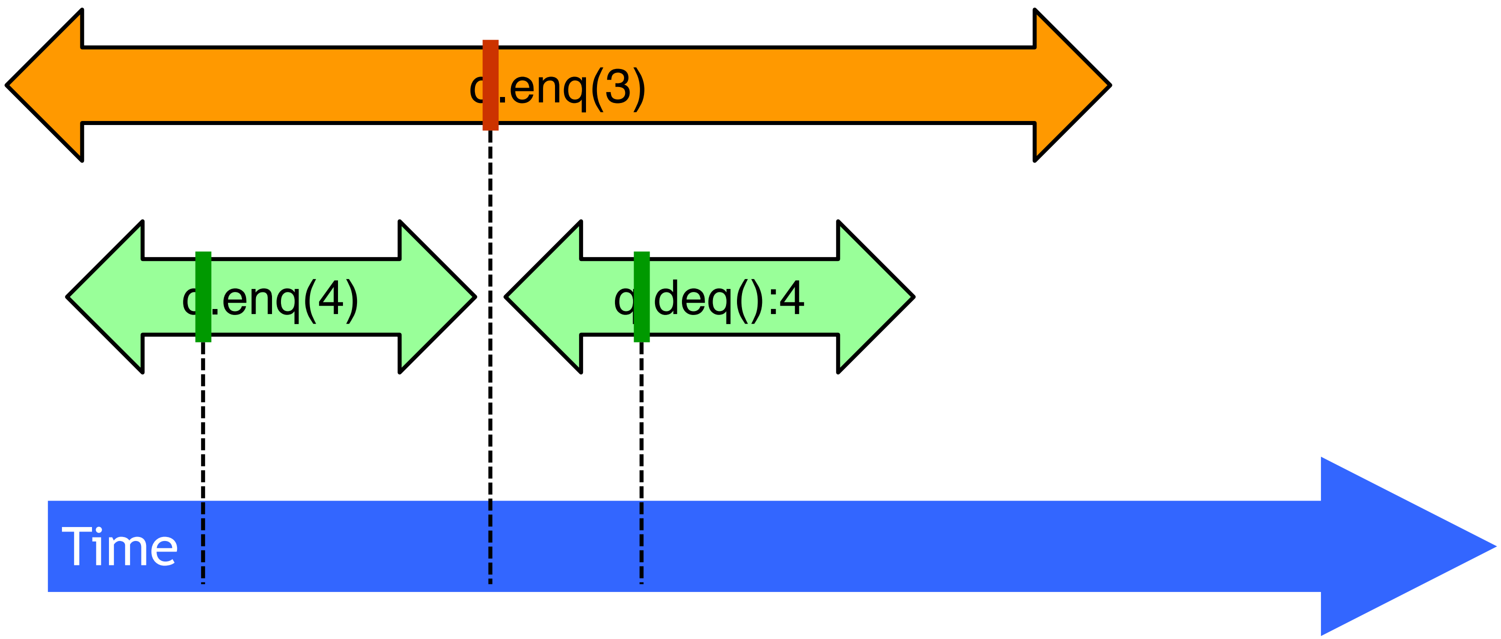

Complete this pending invocation
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
A q:void
Discard this one
Added response
q.enq(3)
q.enq(4)
q.deq():4
q.enq(6)
Time

Example

A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void

Equivalent Sequential history

B q.enq(4)
B q:void
A q.enq(3)
A q:void
B q.deq()
B q:4

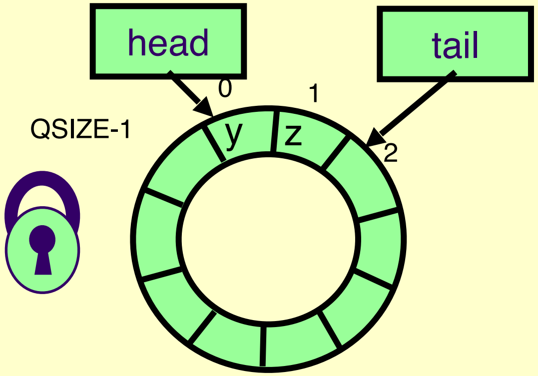


Locality Theorem

- History **H** is linearizable if and only if
  - For every object **x**
  - **H|x** is linearizable
- We care about objects only!
- Why Does Locality Matter?
  - Modularity
  - Can prove linearizability of objects in isolation
  - Can compose independently-implemented objects

Linearizability: Locking

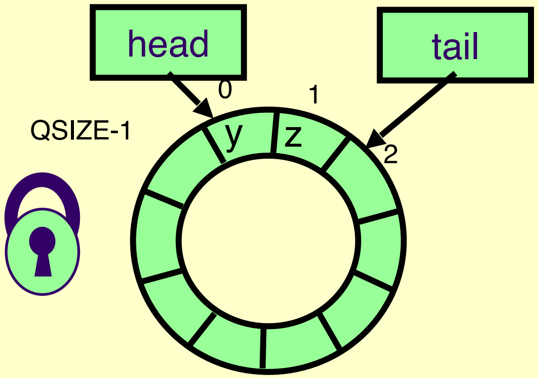
```
public class Queue {
  int head = 0, tail = 0;
  Object[QSIZE] items;

  public synchronized
  void enq(Object x) {
    while (tail – head == QSIZE)
      this.wait();
    items[tail % QSIZE] = x;
    tail++;
    this.notifyAll();
  } …
}
```



As we said, the linearization order is order lock acquired

Linearizability: Lock-free

```
public class LockFreeQueue {
  volatile int head = 0,
               tail = 0;
  Object[QSIZE] items;
  public void enq(Item x) {
    while (tail-head == QSIZE); // busy-wait
    items[tail % QSIZE] = x; tail++;
  }
  public Item deq() {
    while (tail == head); // busy-wait
    Item item = items[head % QSIZE]; head++;
    return item;
  }
}
```



Linearization order is order head and tail fields modified

Strategy

- Identify one atomic step where method "happens"
  - Critical section
  - Machine instruction
- Does not always work
  - Might need to define several different steps for a given method

Alternative: Sequential Consistency

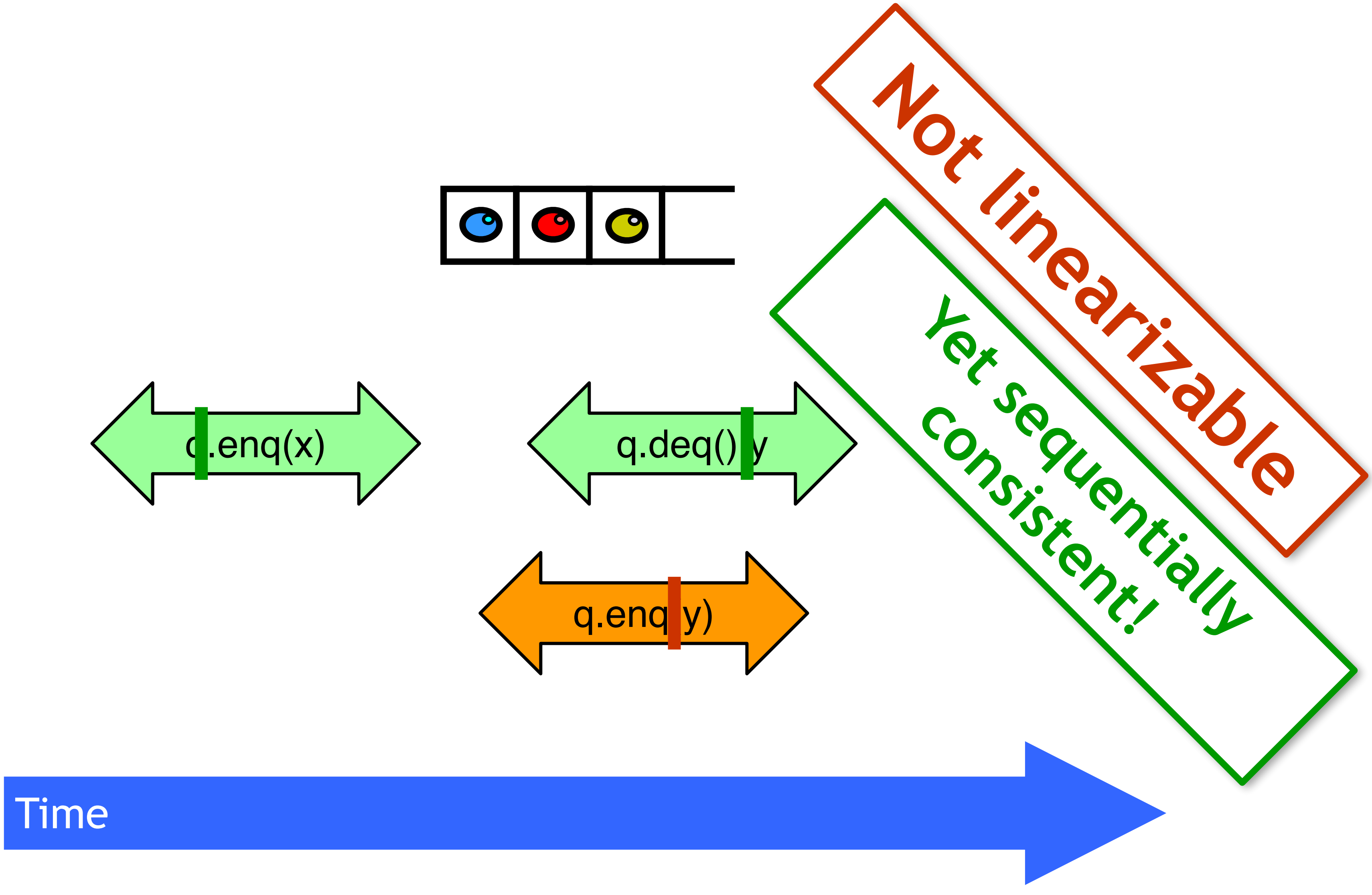
- History **H** is sequentially consistent if it can be extended to **G** by
  - Appending zero or more responses to pending invocations
  - Discarding other pending invocations
  - So that **G** is equivalent to a
  - Legal sequential history **S**
  - ~~Where $\rightarrow_G \subset \rightarrow_S$~~

Differs from linearizability

Alternative: Sequential Consistency

- No need to preserve real-time order
  - Cannot re-order operations done by the same thread (keep program order)
  - Can re-order non-overlapping operations done by different threads
- Often used to describe multiprocessor memory architectures

Example



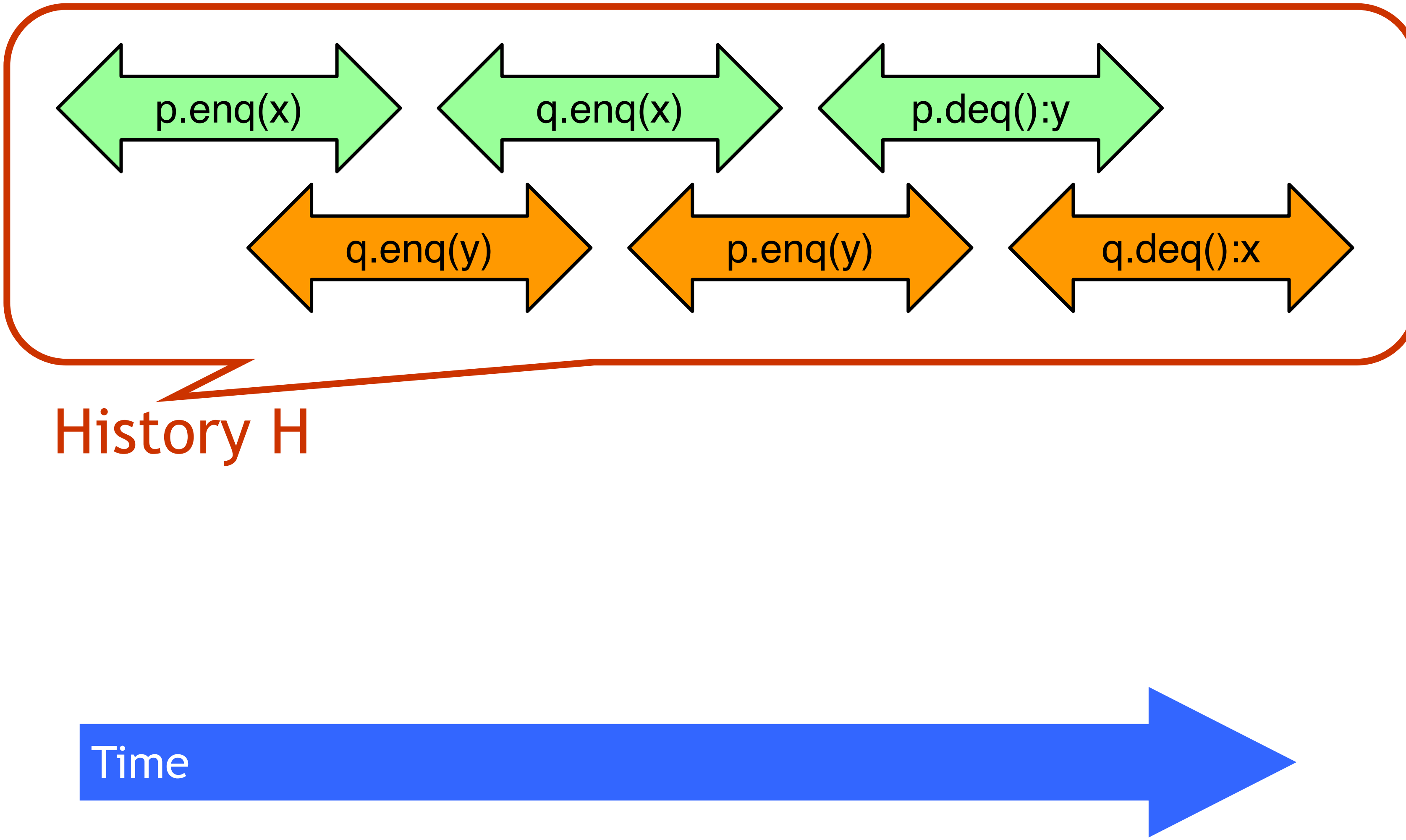
q.enq(x)
q.deq() y
q.enq(y)
Not linearizable
Yet sequentially consistent!
Time

Theorem

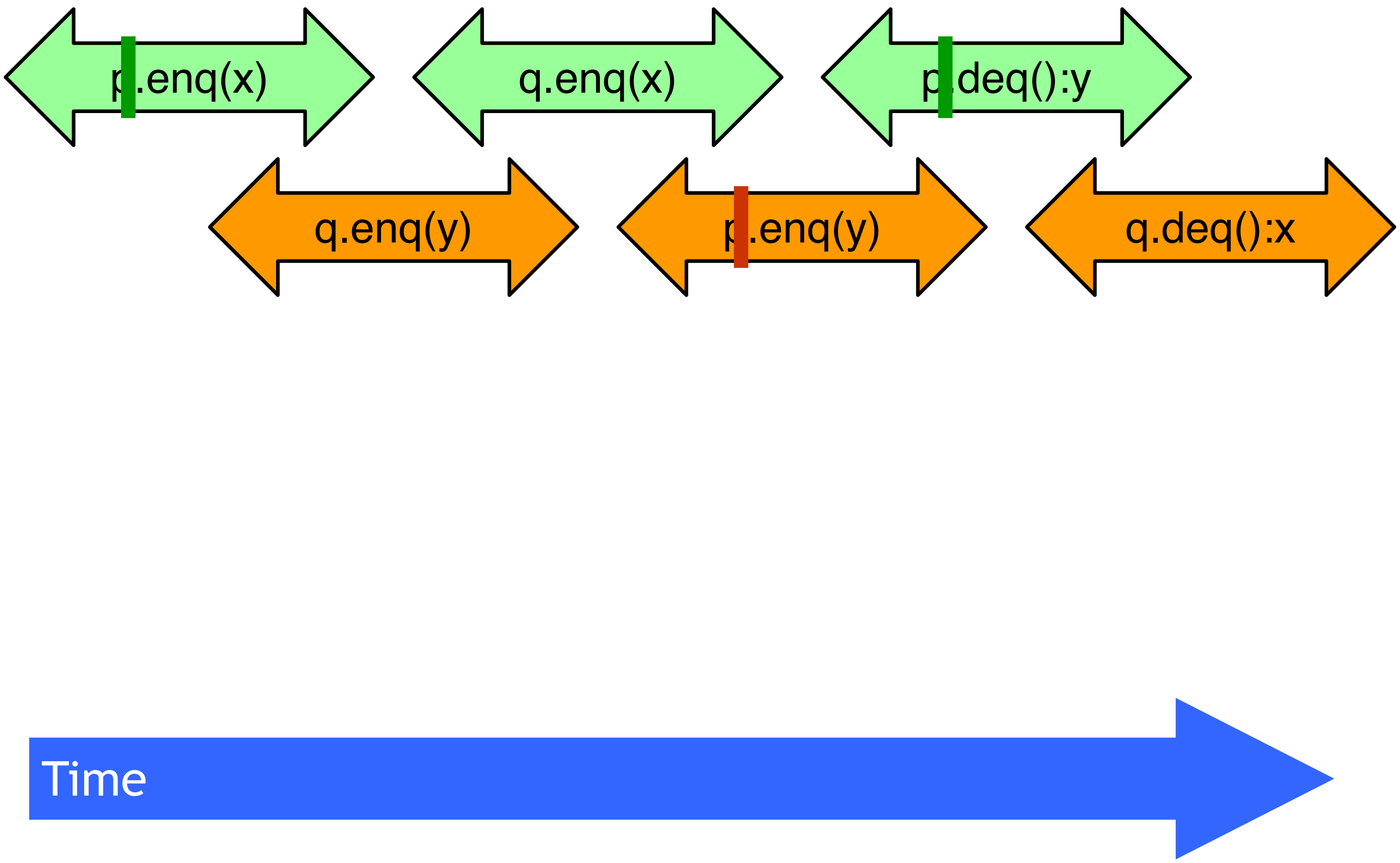
- Sequential consistency is not a local property

  (and thus we lose composability…)

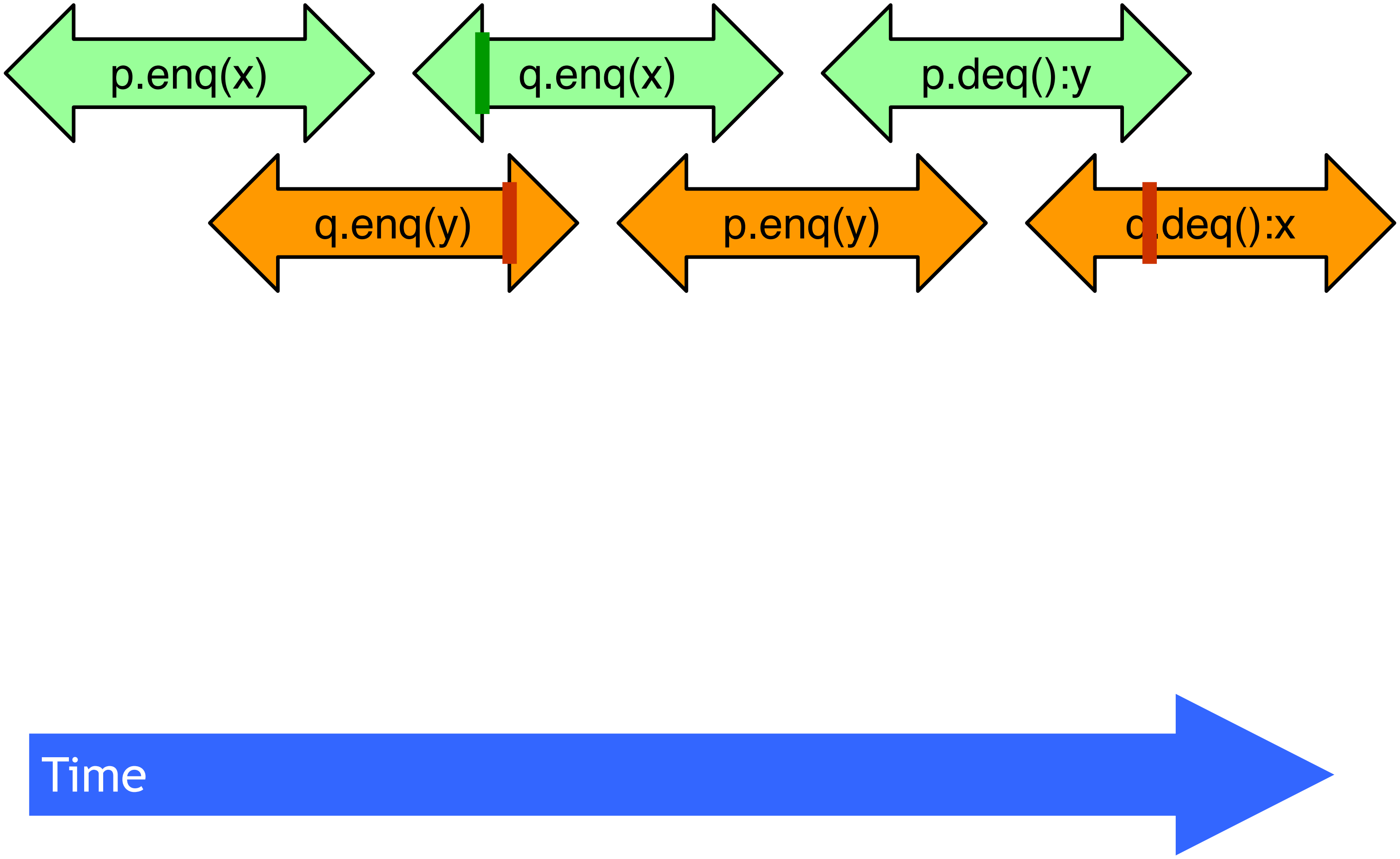
FIFO Queue Example



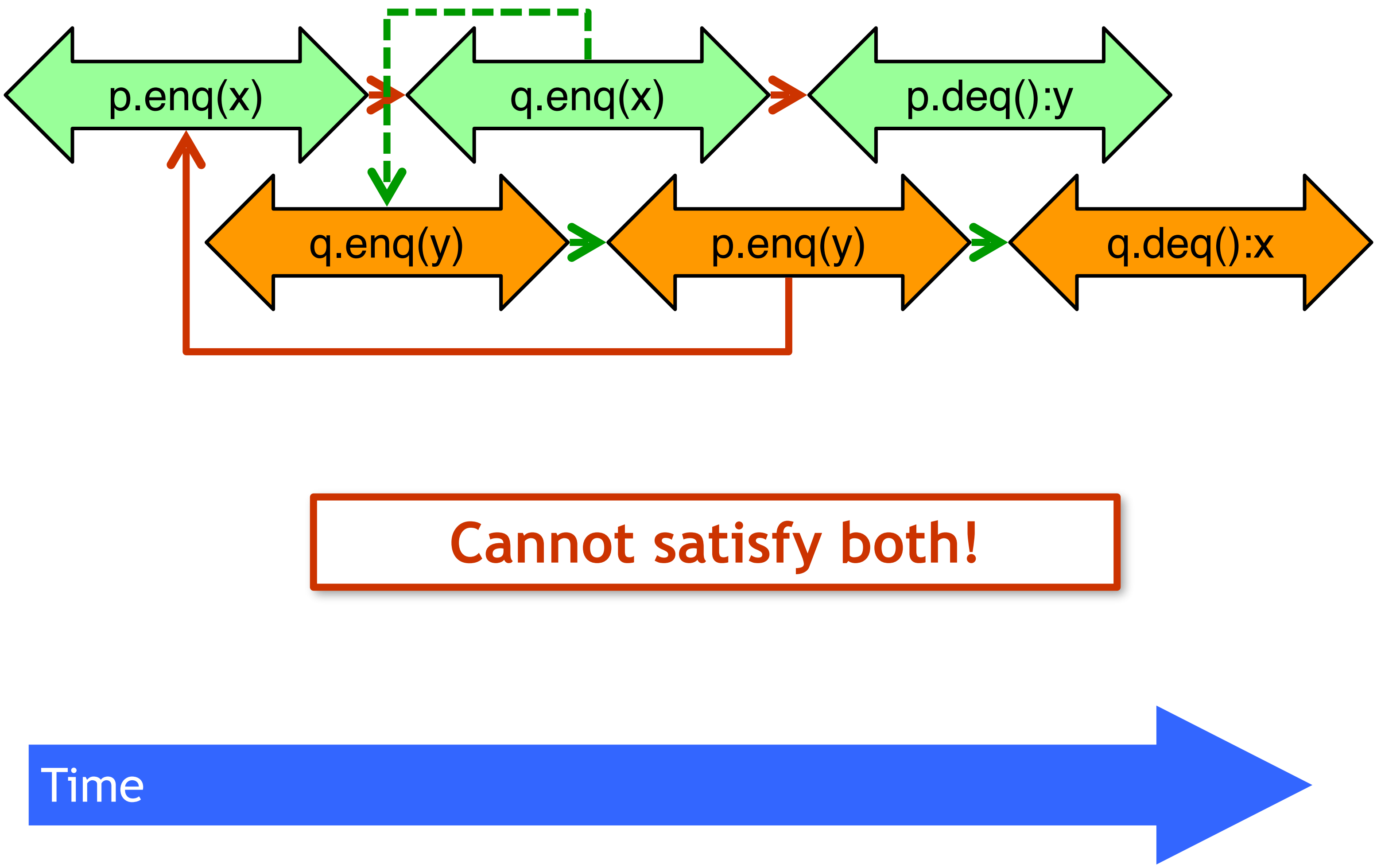
H|p Sequentially Consistent



H|q Sequentially Consistent



Ordering Imposed by p and q

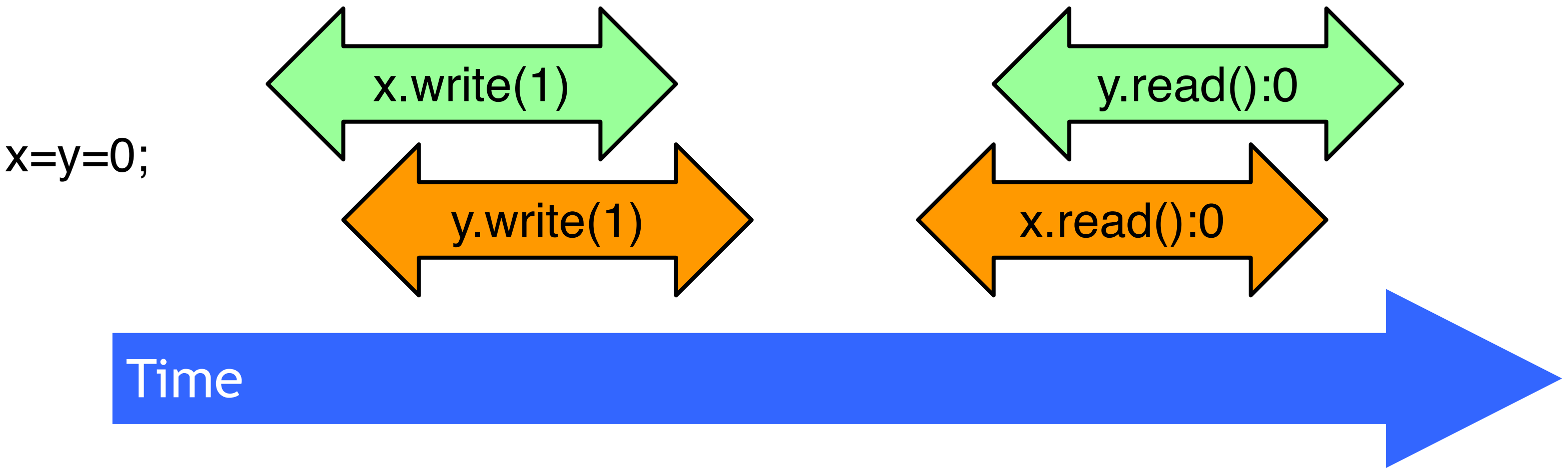


Fact

- Most hardware architectures do not support sequential consistency
- Because they think it is too strong
- Here is another story…

The Flag Example

- Each thread's view is sequentially consistent
  - It went first
- Entire history is not sequentially consistent
  - Cannot both go first
- Is this behavior really so wrong?



Opinion 1: It is Wrong!

- This pattern
  - Write mine, read yours
- Is exactly the flag principle
  - Beloved of Alice and Bob
  - Heart of mutual exclusion
    - Peterson
    - Bakery, etc.
- It is non-negotiable!

Opinion 2: But It Feels So Right…

- Many hardware architects think that sequential consistency is too strong
- Too expensive to implement in modern hardware
- OK if flag principle
  - Violated by default
  - Honoured by explicit request

Memory Hierarchy

- On modern multiprocessors, processors do not read and write directly to memory
- Memory accesses are very slow compared to processor speeds
- Instead, each processor reads and writes directly to a cache
  - To read a memory location: load data into cache and read from cache
  - To write a memory location: update cached copy and lazily write cached data back to memory

While Writing to Memory

- A processor can execute hundreds, or even thousands of instructions
- Why delay on every memory write?
- Instead, write back in parallel with rest of the program

Revisionist History

- Flag violation history is actually OK
    - Processors delay writing to memory
    - Until after reads have been issued
- Otherwise unacceptable delay between read and write instructions
- Who knew you wanted to synchronize?

Synchronizing

- Writing to memory = mailing a letter
- Vast majority of reads & writes
    - Not for synchronization
    - No need to idle waiting for post office
- If you want to synchronize
    - Announce it explicitly
    - Pay for it only when you need it

Explicit Synchronization

- Memory barrier instruction
  - Flush unwritten caches
  - Bring caches up to date
- Compilers often do this for you
  - Entering and leaving critical sections
- Expensive

Volatile

- In Java, can ask compiler to keep a variable up-to-date with volatile keyword
- Also inhibits reordering, removing from loops, & other "optimizations"

Real-World Hardware Memory

- Weaker than sequential consistency
- But you can get sequential consistency at a price
- OK for experts, tricky stuff
  - Assembly language, device drivers, etc.
  - Know your architecture
- Linearizability more appropriate for high-level software

Critical Sections

- Easy way to implement linearizability
  - Take sequential object
  - Make each method a critical section
- Like synchronized methods in Java
- Problems
  - Blocking
  - No concurrency

Summary

- Linearizability
  - Powerful specification tool for shared objects
  - Allows us to capture the notion of objects being "atomic"
  - Operation takes effect instantaneously between invocation and response
  - Uses sequential specification, locality implies composablity
  - Good for high level objects

Summary

- Sequential Consistency
  - Not composable
  - Harder to work with
  - Good way to think about hardware models
- We will use **linearizability** in the remainder of this course unless stated otherwise